1. Prove that the number $\sqrt{2}$ is irrational.

2. Prove that the number $\log_2(3)$ is irrational.

3. Let $x \in \mathbb{R}$ satisfy $x^7 + 5x^2 - 3 = 0$. Then prove that $x$ is irrational.

4. Let $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then $a$ or $b$ is even.

5. Prove that if $k$ is a positive integer and $\sqrt{k}$ is not an integer, then $\sqrt{k}$ is irrational.

6. Let $n \in \mathbb{N}$, $n \geq 2$, and $a, b, c \in \mathbb{Z}$. Prove that if $ab \equiv 1 \pmod{n}$, then $\forall c \neq 0 \mod{n}$ we have $ac \neq 0 \mod{n}$.

7. Prove that there do not exist $a, n \in \mathbb{N}$ such that $a^2 + 35 = 7^n$.

8. Let $A, B$ be nonempty finite sets and assume that there is a bijection, $f$, from $A$ to $B$. Then prove that if $g : A \to B$ is an injective function, then it is surjective. Would this statement be still true if the sets were not finite?

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function with $f''(x) > 0$. Prove that $f$ cannot have 3 zeroes.

10. Let $(x_n)_{n \in \mathbb{N}}$ be a real sequence. Then, recall that we say $(x_n)$ converges to $L$ if

$$\forall \epsilon > 0, \exists N \in \mathbb{N}, \forall n \geq N, |x_n - L| < \epsilon.$$ 

Prove that if a sequence $(y_n)$ converges, then the limit is unique.