1. Consider the function $f : \{1, 2, 3, 4, 5, 6, 7\} \to \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ given as
   
   \[ f = \{(1,3), (2,8), (3,3), (4,1), (5,2), (6,4), (7,6)\}. \]

   Find: $f(\{1,2,3\})$, $f(\{4,5,6,7\})$, $f(\emptyset)$, $f^{-1}(\{0,5,9\})$ and $f^{-1}(\{0,3,5,9\})$.

2. Find counterexamples to the following
   
   (a) Given a function $f : A \to B$ and subsets $W, X \subseteq A$, we have $f(W \cap X) = f(W) \cap f(X)$.
   (b) Given a function $f : A \to B$ and a subset $Y \subseteq B$, we have $f(f^{-1}(Y)) = Y$.

3. Consider $f : A \to B$. Prove that $f$ is injective if and only if $X = f^{-1}(f(X))$ for all $X \subseteq A$.

4. Consider $f : A \to B$. Prove that $f$ is surjective if and only if $f(f^{-1}(Y)) = Y$ for all $Y \subseteq B$.

5. Let $A$ and $B$ be nonempty sets. Prove that if $f$ is an injection, then $f(A - B) = f(A) - f(B)$.

6. Let $A = \{a_1, a_2, a_3, \ldots, a_n\}$ be a nonempty set of $n$ distinct natural numbers. Prove that there exists a nonempty subset of $A$ for which the sum of its elements is divisible by $n$.
   
   **Hint:** Consider the sums $s_k = a_1 + a_2 + \cdots + a_k$.

7. Let $n \in \mathbb{N}$ with $n > 1$ and let $P$ be the set of polynomials with coefficients in $\mathbb{R}$.
   
   (a) We define a relation, $T$, on $P$ as follows:
   
   Let $f, g \in P$. Then we say $fTg$ if $f - g = c$ for some $c \in \mathbb{R}$. Show that $T$ is an equivalence relation on $P$.
   (b) Let $\mathcal{R}$ be the set of equivalence classes of $P$ and let $F : \mathcal{R} \to P$ be the derivative operator defined as $F([f]) = \frac{df}{dx}$. Is $F$ well defined (i.e. is it a function)? Is it surjective? Is it injective?

8. **Definition:** A set $A$ is finite if there exists a nonnegative integer $c$ such that there exists a bijection from $A$ to $\{n \in \mathbb{N} : n \leq c\}$. (The integer $c$ is called the cardinality of $A$.)
   
   (a) Let $A$ be a finite set, and let $B$ be a subset of $A$. Prove that $B$ is finite. (Hint: induction on $|A|$. Note that our proof can’t use induction on $|B|$, or indeed refer to “the number of elements in $B$” at all, because we don’t yet know that $B$ is finite!)
   (b) Prove that the union of two disjoint finite sets is finite.
   (c) Prove that the union of any two finite sets is finite. (Hint: $A \cup B = A \cup (B - A)$)