1. (a) Write the converse, contrapositive and negation of the following statement:

   For every integer $n$, if $n$ is divisible by 3 then $n^2$ is divisible by 3.

   (b) Let $A_n$ be the interval $[-n\ ,\ 2+\frac{4}{n^3})$ for $n \in \mathbb{N}$. Find $\bigcap_{n\in\mathbb{N}} A_n$ and $\bigcup_{n\in\mathbb{N}} A_n$ (no proof is required).

   (c) Write $\mathbb{R} - \mathbb{N}$ as a union of an indexed collection of sets where each set is an interval.

2. Consider the following two statements:

   1. $\forall n \in \mathbb{N}$, $\exists\, z \in \mathbb{Z}$ such that $z = n$
   2. $\exists\, z \in \mathbb{Z}$ such that $\forall n \in \mathbb{N}$, $z = n$

   One of the statements is true, and the other is false. Determine which is which and prove both of your answers.

3. (a) Give the definition of the power set $\mathcal{P}(A)$ of a set $A$.

   (b) Let $A = \{1, 2, \{1, 2\}\}$. Determine whether the following statements are True or False (and provide a brief explanation why).

   (a) $\{1, 2\} \subseteq A$.
   (b) $\{1, 2\} \subseteq \mathcal{P}(A)$.
   (c) $\{1, 2\} \in A$.
   (d) $\{1, 2\} \in \mathcal{P}(A)$.

4. Let $A := \{n \in \mathbb{N} : \exists\, z \in \mathbb{Z} \text{ such that } n = 2z + 1\}$ and let $B := \{n \in \mathbb{N} : \exists\, k \in \mathbb{N} \text{ such that } n = 2k\}$. Determine the following:

   1. $A \cap B$,
   2. $A \cup B$,
   3. $A - B$, and
   4. $B - A$.

5. True or False: You do not have to give explanations.

   (a) $\mathbb{N} \subset \mathbb{Z}$.
   (b) $[2, 3] \in \mathcal{P}(\mathbb{R})$.
   (c) $2 \in \mathcal{P}(\mathbb{R})$.
   (d) $\cup_{n\in\mathbb{Z}}(n, n + 2) = \mathbb{R}$.
   (e) $\mathbb{Q} \cap (\sqrt{2}, \infty) = \mathbb{Q} \cap [\sqrt{2}, \infty)$.

6. Using any method you like, prove that the following statements are logically equivalent:

   Statement 1: $Q \Rightarrow (R \Rightarrow S)$
   Statement 2: $(Q \wedge R) \Rightarrow S$.

7. Let $n \in \mathbb{Z}$. Prove that $n^3 - 5n^2 + 13$ is odd.

8. (a) Write the negation of the following statement:
   "For every positive $\epsilon$ there exists a positive $\delta$ such that if $|x| < \delta$ then $|f(x)| < \epsilon$."
   (b) Is the number $0.7(\pi - 3.1415)$ rational or irrational? (Include a short proof; you can assume without proof that $\pi$ is irrational).

9. Prove that if $n$ and $m$ are odd integers, then $n^2 - m^2 \equiv 0 \pmod 8$ .

10. (a) Prove that $\sqrt{10}$ is irrational.
    (b) Prove that the following statement is False:
    If $x, y$ are both irrational, then $x - y$ is irrational.
    (c) Prove that $\sqrt{5} - \sqrt{2}$ is irrational.

11. (a) Write the negation of the following statement:
    "For every $(a, b) \in \mathbb{N} \times \mathbb{N}$, if $a > b$ then $(a + b)^2 \geq (a - b)^2$."
    (b) Write the converse and contrapositive of the following statement:
    "If it is raining outside then this is Vancouver."
    (c) Give a precise mathematical definitions of the following sets

$$B = \bigcup_{\alpha \in I} S_\alpha \qquad\qquad C = \bigcap_{\alpha \in I} S_\alpha$$

    (d) For any $n \in \mathbb{N}$, let $A_n = \left(\frac{1}{n} - 1, 3 + \frac{1}{n^2}\right]$. Simplify the following sets

$$B = \bigcup_{n \in \mathbb{N}} A_n \qquad\qquad C = \bigcap_{n \in \mathbb{N}} A_n$$

    (e) Let $A, B$ be sets in some universal set $U$. Suppose that $\bar{A} = \{3, 8, 9\}$, $A - B = \{1, 2\}$, $B - A = \{8\}$ and $A \cap B = \{5, 7\}$ Determine $A, B, U$.

12. (a) Prove or disprove the following statement
    Let $a, b, c, d \in \mathbb{R}$. If $ab \geq cd$ then $a \geq c$ and $b \geq d$.
    (b) Prove or disprove the following
    Let $a, b, c, n \in \mathbb{Z}$ so that $n \geq 3$. If $a + b \equiv 1 \mod n$ and $b + c \equiv 1 \mod n$ then $a + c \equiv 2 \mod n$.
    (c) Repeat part (b), but when $n = 2$.

13. Let $n \in \mathbb{Z}$. Prove that $n^2 + 1$ is odd if and only if $7n + 3$ is odd.

14. Determine whether the following four statements are true or false — explain your answers ("true" or "false" is not sufficient).
    (i) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}$, if $(xy \geq 0)$ then $(x + y \geq 0)$.
    (ii) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. if $(xy \geq 0)$ then $(x + y \geq 0)$.
    (iii) $\exists x \in \mathbb{R}$ s.t. $\exists y \in \mathbb{R}$ s.t. if $(xy \geq 0)$ then $(x + y \geq 0)$.