1. (a) Write the converse, contrapositive and negation of the following statement:
   For every integer \( n \), if \( n \) is divisible by 3 then \( n^2 \) is divisible by 3.
   
   (b) Let \( A_n \) be the interval \([-n, 2 + \frac{4}{n^2})\) for \( n \in \mathbb{N} \). Find \( \bigcap_{n \in \mathbb{N}} A_n \) and \( \bigcup_{n \in \mathbb{N}} A_n \) (no proof is required).
   
   (c) Write \( \mathbb{R} - \mathbb{N} \) as a union of an indexed collection of sets where each set is an interval.

2. Consider the following two statements:
   1. \( \forall n \in \mathbb{N}, \exists z \in \mathbb{Z} \text{ such that } z = n \)
   2. \( \exists z \in \mathbb{Z} \text{ such that } \forall n \in \mathbb{N}, z = n \)

   One of the statements is true, and the other is false. Determine which is which and prove both of your answers.

3. (a) Give the definition of the power set \( \mathcal{P}(A) \) of a set \( A \).
   
   (b) Let \( A = \{1, 2, \{1, 2\}\} \). Determine whether the following statements are True or False (and provide a brief explanation why).
      
      (a) \( \{1, 2\} \subseteq A \).
      (b) \( \{1, 2\} \subseteq \mathcal{P}(A) \).
      (c) \( \{1, 2\} \in A \).
      (d) \( \{1, 2\} \in \mathcal{P}(A) \).

4. Let \( A := \{n \in \mathbb{N} : \exists z \in \mathbb{Z} \text{ such that } n = 2z + 1\} \) and let \( B := \{n \in \mathbb{N} : \exists k \in \mathbb{N} \text{ such that } n = 2k\} \). Determine the following:
   1. \( A \cap B \),
   2. \( A \cup B \),
   3. \( A - B \), and
   4. \( B - A \).

5. True or False: You do not have to give explanations.
   
   (a) \( \mathbb{N} \subset \mathbb{Z} \).
   (b) \([2, 3] \in \mathcal{P}(\mathbb{R})\).
   (c) \( 2 \in \mathcal{P}(\mathbb{R}) \).
   (d) \( \bigcup_{n \in \mathbb{Z}} (n, n + 2) = \mathbb{R} \).
   (e) \( \mathbb{Q} \cap (\sqrt{2}, \infty) = \mathbb{Q} \cap [\sqrt{2}, \infty) \).

6. Using any method you like, prove that the following statements are logically equivalent:
   
   Statement 1: \( Q \Rightarrow (R \Rightarrow S) \),
   Statement 2: \( (Q \land R) \Rightarrow S \).

7. Let \( n \in \mathbb{Z} \). Prove that \( n^3 - 5n^2 + 13 \) is odd.
8. (a) Write the negation of the following statement:
   “For every positive $\epsilon$ there exists a positive $\delta$ such that if $|x| < \delta$ then $|f(x)| < \epsilon$.”

   (b) Is the number $0.7(\pi - 3.1415)$ rational or irrational? (Include a short proof; you can assume without proof that $\pi$ is irrational).

9. Prove that if $n$ and $m$ are odd integers, then $n^2 - m^2 \equiv 0 \pmod{8}$.

10. (a) Prove that $\sqrt{10}$ is irrational.

    (b) Prove that the following statement is False:
        If $x, y$ are both irrational, then $x - y$ is irrational.

    (c) Prove that $\sqrt{5} - \sqrt{2}$ is irrational.

11. (a) Write the negation of the following statement:
    “For every $(a, b) \in \mathbb{N} \times \mathbb{N}$, if $a > b$ then $(a + b)^2 \geq (a - b)^2$.”

    (b) Write the converse and contrapositive of the following statement:
        “If it is raining outside then this is Vancouver.”

    (c) Give a precise mathematical definitions of the following sets
        \[
        B = \bigcup_{\alpha \in I} S_{\alpha} \quad \quad \quad C = \bigcap_{\alpha \in I} S_{\alpha}
        \]

    (d) For any $n \in \mathbb{N}$, let $A_n = \left(\frac{1}{n} - 1, 3 + \frac{1}{n^2}\right]$. Simplify the following sets
        \[
        B = \bigcup_{n \in \mathbb{N}} A_n \quad \quad \quad C = \bigcap_{n \in \mathbb{N}} A_n
        \]

    (e) Let $A, B$ be sets in some universal set $U$. Suppose that $\bar{A} = \{3, 8, 9\}$, $A - B = \{1, 2\}$, $B - A = \{8\}$ and $A \cap B = \{5, 7\}$ Determine $A, B, U$.

12. (a) Prove or disprove the following statement
    Let $a, b, c, d \in \mathbb{R}$. If $ab \geq cd$ then $a \geq c$ and $b \geq d$.

    (b) Prove or disprove the following
    Let $a, b, c, n \in \mathbb{Z}$ so that $n \geq 3$. If $a + b \equiv 1 \pmod{n}$ and $b + c \equiv 1 \pmod{n}$ then $a + c \equiv 2 \pmod{n}$.

    (c) Repeat part (b), but when $n = 2$.

13. Let $n \in \mathbb{Z}$. Prove that $n^2 + 1$ is odd if and only if $7n + 3$ is odd.

14. Determine whether the following four statements are true or false — explain your answers (“true” or “false” is not sufficient).
   (i) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R},$ if $(xy \geq 0)$ then $(x + y \geq 0)$.
   (ii) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. if $(xy \geq 0)$ then $(x + y \geq 0)$.
   (iii) $\exists x \in \mathbb{R}$ s.t. $\exists y \in \mathbb{R}$ s.t. if $(xy \geq 0)$ then $(x + y \geq 0)$. 