Homework 7

- Chapter 9: Questions 10, 16, 18, 24, 28, 34. Note that these are all “prove or disprove” questions: each statement could be true or false.

- Let $S$ be a nonempty subset of $\{n \in \mathbb{Z}: n \leq 0\}$. Use the Well-Ordering Principle to show that $S$ has a greatest element; in other words, prove there exists $s \in S$ such that $s \geq x$ for all $x \in S$.

- Use the Well-Ordering Principle to show that every integer greater than 1 has a prime divisor. (Hint: consider the set $B = \{n \in \mathbb{N}, n > 1: n$ does not have a prime divisor}. Does $B$ have a least element?)

- (a) Prove that the set $\{x \in \mathbb{Q}: 0 < x < 1\}$ is not well-ordered.
  (b) Prove or disprove the statement: the set $\{x \in \mathbb{Q}: 0 \leq x \leq 1\}$ is well-ordered.