Homework 10

- Section 12.1: Questions 10, 12.
- Section 12.2: Questions 4, 6, 16
  (Hint: For question 16, to count the number of injective function, first consider how many different options you have for \( f(A) \) and fixing the value of \( f(A) \), find how many different options you have for \( f(B) \), and so on to send all elements in the domain to an element in the codomain.)
- Section 12.3: Question 2
- Section 12.4: Questions 8, 10

Using induction on the size of \( A \), prove that if \( A \) and \( B \) are finite sets such that \(|A| > |B|\) and \( f : A \to B \) is a function, then \( f \) is not injective.

- For \( n \in \mathbb{N} \), let \( A = \{a_1, a_2, a_3, \ldots, a_n\} \) be a set and let \( F \) be the set of all functions \( f : A \to \{0, 1\} \) from \( A \) to \( \{0, 1\} \). What is the size of \( F \)?
  Now, for \( \mathcal{P}(A) \), the power set of \( A \), consider the function \( g : F \to \mathcal{P}(A) \), defined as
  \[
g(f) = \{a \in A : f(a) = 1\}.
\]
  Is \( g \) injective? Is \( g \) surjective?