Homework 8 **Due: November 3 (Friday)**

1. Chapter 10: Question 8
2. Chapter 10: Question 12
3. Chapter 10: Question 18
4. Chapter 10: Question 22
5. The *Fibonacci numbers* are defined to be $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$. Show that for all $k \in \mathbb{N}$, F_{4k} is a multiple of 3. (Hint: in the inductive step, you may need to use the definition of Fibonacci number a few times in order to apply the induction hypothesis.)
6. Use induction to prove that if A is a finite set of cardinality $n \geq 0$ then $|\mathcal{P}(A)| = 2^n$. (Hint: for the inductive step, it is useful to consider the fact that any subset of a set C of $k+1$ elements either contains the last element of C or does not contain it.)
7. Let $f(x) = x \ln x$ and $x > 0$. Denote $f^{(n)}(x)$ the nth derivative of $f(x)$ for $n \in \mathbb{N}$. Prove that for all integer $n \geq 3$ it holds
$$f^{(n)}(x) = (-1)^n \frac{(n-2)!}{x^{n-1}}.$$
8. A sequence $a_1, a_2, ..., a_n, ...$ is defined by
$$a_1 = 1, a_2 = 4, \text{ and } a_n = 2a_{n-1} - a_{n-2} + 2 \text{ for all } n \geq 3.$$
Show: $a_n = n^2$ for all $n \in \mathbb{N}$. (Hint: use strong induction.)
9. Suppose you begin with a pile of n stones ($n \geq 2$) and split this pile into n piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile you multiply the number of stones in each of the two smaller piles you form, so that if these piles have p and q stones in them, respectively, you compute pq. Show that no matter how you split the piles (eventually into n piles of one stone each), the sum of the products computed at each step equals $n(n-1)/2$. (Hint: use strong induction on n.)