1. Section 1.5: #4 (a)-(h),

Solution:

We are given: $A = \{b, c, d\}, B = \{a, b\}$. Some simpler sets that appear in the questions:

$A \times B = \{(b, a), (b, b), (c, a), (c, b), (d, a), (d, b)\}$

$B \times B = \{(a, a), (a, b), (b, a), (b, b)\}$

$A \cap B = \{b\}$

$P(A) = \{\emptyset, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$

$P(B) = \{\emptyset, \{a\}, \{b\}\{a, b\}\}$

Now we have

(a) $(A \times B) \cap (B \times B) = \{(b, a), (b, b)\}$.

(b) $(A \times B) \cup (B \times B) = \{(b, a), (b, b), (c, a), (c, b), (d, a), (d, b), (a, a), (a, b)\}$.

(c) $(A \times B) - (B \times B) = \{(c, a), (c, b), (d, a), (d, b)\}$.

(d) $(A \cap B) \times A = \{(b, b), (b, c), (b, d)\}$.

(e) $(A \times B) \cap B = \emptyset$.

(f) $P(A) \cap P(B) = \{\emptyset, \{b\}\}$.

(g) $P(A) - P(B) = \{\{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$.

(h) $P(A \cap B) = \{\emptyset, \{b\}\}$.

2. Section 1.6: Do problem #2, but use $B = \{1, 2, 3, 4\}$ instead of the set B listed in the problem (keep A and U the same).

Solution: (a) $\overline{A} = \{1, 3, 5, 7\}$

(b) $\overline{B} = \{0, 5, 6, 7, 8\}$

(c) $A \cap \overline{A} = \emptyset$

(d) $A \cup \overline{A} = U = \{0, 1, ..., 8\}$

(e) $A - \overline{A} = A = \{0, 2, 4, 6, 8\}$

(f) $\overline{A \cup B} = \{5, 7\}$

(g) $\overline{A} \cap \overline{B} = \{5, 7\}$

(h) $\overline{A \cap B} = \{0, 1, 3, 5, 6, 7, 8\}$

(i) $\overline{A} \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2), (3, 3), (3, 4),$
$(5, 1), (5, 2), (5, 3), (5, 4), (7, 1), (7, 2), (7, 3), (7, 4)\}$

3. Section 1.7: #12, #14

Solution: #12: $(A - B) \cup (C \cap B)$, or $(A \cup B) - (B - C)$
#14: $(A - (B \cup C)) \cup (A \cap B \cap C)$
Note: there are other possible correct answers for both problems.

4. Section 1.8: #4

Solution: (a) $\bigcup_{i \in \mathbb{N}} A_i = \{..., -6, -4, -2, 0, 2, 4, 6, ...\} = \{2n : n \in \mathbb{Z}\}$
(b) $\bigcap_{i \in \mathbb{N}} A_i = \{0\}$

5. Section 1.8: #6

Solution: (a)

$$\bigcup_{i \in \mathbb{N}} [0, i+1] = \{x : x \in [0, i+1], \text{for some } i \in \mathbb{N}\} = [0, \infty)$$

(b)

$$\bigcap_{i \in \mathbb{N}} [0, i+1] = \{x : x \in [0, i+1], \text{for every } i \in \mathbb{N}\} = [0, 2]$$

6. Section 1.8: #8

Solution: For each $\alpha \in \mathbb{R}$, the set $\{\alpha\} \times [0, 1] = \{(\alpha, y) : y \in [0, 1]\}$ is the vertical line segment connecting the points $(\alpha, 0)$ and $(\alpha, 1)$ in the plane $\mathbb{R}^2$. For (a), the union of all these line segments is the horizontal strip:

$$\bigcup_{\alpha \in \mathbb{R}} \{\alpha\} \times [0, 1] = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq 1\} = \mathbb{R} \times [0, 1].$$

For (b), the intersection of all these line segments is empty (even the intersection of two such line segments is already empty):

$$\bigcap_{\alpha \in \mathbb{R}} \{\alpha\} \times [0, 1] = \emptyset.$$