Math 220. Homework 2. Due Friday September 23.

- **Section 1.5: Problem 4.** We are given: $A = \{b, c, d\}$, $B = \{a, b\}$. Before answering all the questions, we compute some of the simpler sets that appear:

$$A \times B = \{(b, a), (b, b), (c, a), (c, b), (d, a), (d, b)\};$$
$$B \times B = \{(a, a), (a, b), (b, a), (b, b)\};$$
$$A \cap B = \{b\};$$
$$\mathcal{P}(A) = \{\varnothing, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\};$$
$$\mathcal{P}(B) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}.$$

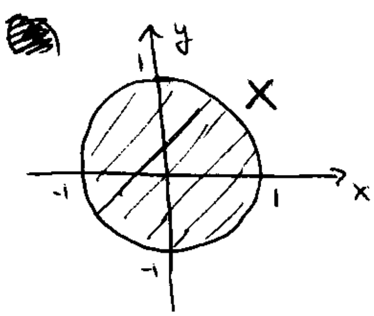
Now we are ready to answer all the questions.

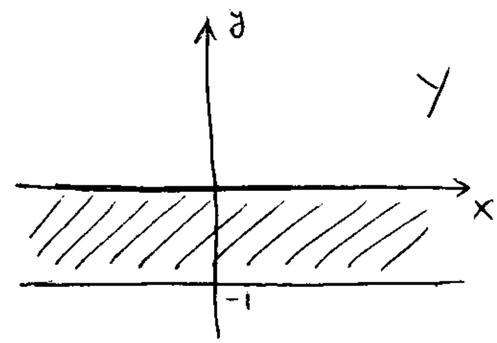
(a) $(A \times B) \cap (B \times B) = \{(b, a), (b, b)\}$.
(b) $(A \times B) \cup (B \times B) = \{(b, a), (b, b), (c, a), (c, b), (d, a), (d, b), (a, a), (a, b)\}$.
(c) $(A \times B) - (B \times B) = \{(c, a), (c, b), (d, a), (d, b)\}$.
(d) $(A \cap B) \times A = \{(b, b), (b, c), (b, d)\}$.
(e) $(A \times B) \cap B = \varnothing$.
(f) $\mathcal{P}(A) \cap \mathcal{P}(B) = \{\varnothing, \{b\}\}$.
(g) $\mathcal{P}(A) - \mathcal{P}(B) = \{\{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$.
(h) The elements of this set are going to be, by definition, pairs of sets, and there will be $8 \cdot 4 = 32$ of them. Here they are:

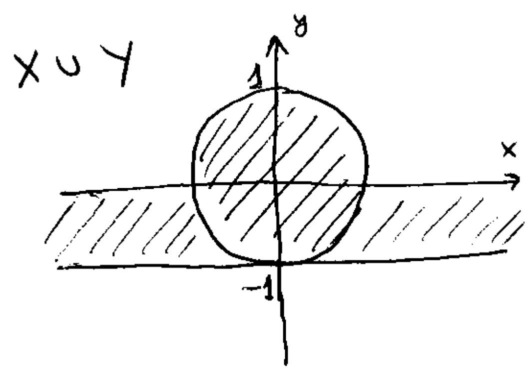
$\mathcal{P}(A) \times \mathcal{P}(B) =$
$\{(\varnothing, \varnothing), (\{b\}, \varnothing), (\{c\}, \varnothing), (\{d\}, \varnothing), (\{b, c\}, \varnothing), (\{b, d\}, \varnothing), (\{c, d\}, \varnothing), (\{b, c, d\}, \varnothing),$
$(\varnothing, \{a\}), (\{b\}, \{a\}), (\{c\}, \{a\}), (\{d\}, \{a\}), (\{b, c\}, \{a\}), (\{b, d\}, \{a\}), (\{c, d\}, \{a\}), (\{b, c, d\}, \{a\}),$
$(\varnothing, \{b\}), (\{b\}, \{b\}), (\{c\}, \{b\}), (\{d\}, \{b\}), (\{b, c\}, \{b\}), (\{b, d\}, \{b\}), (\{c, d\}, \{b\}), (\{b, c, d\}, \{b\}),$
$(\varnothing, \{a, b\}), (\{b\}, \{a, b\}), (\{c\}, \{a, b\}), (\{d\}, \{a, b\}), (\{b, c\}, \{a, b\}), (\{b, d\}, \{a, b\}), (\{c, d\}, \{a, b\}), (\{b, c, d\}, \{a, b\}$

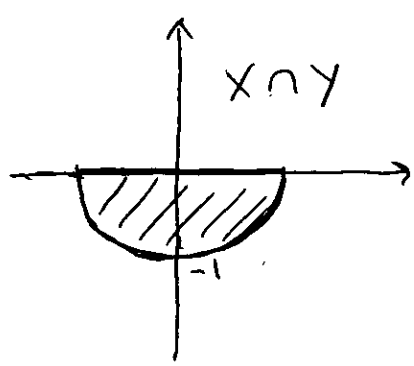
- **Section 1.5, Problem 8.**

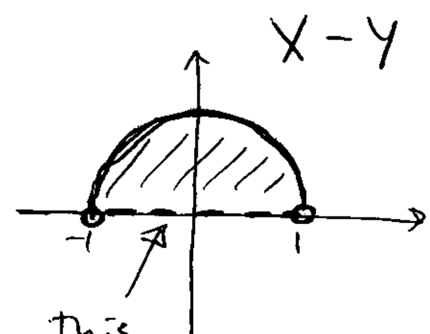
All boundary points are included unless otherwise specified.





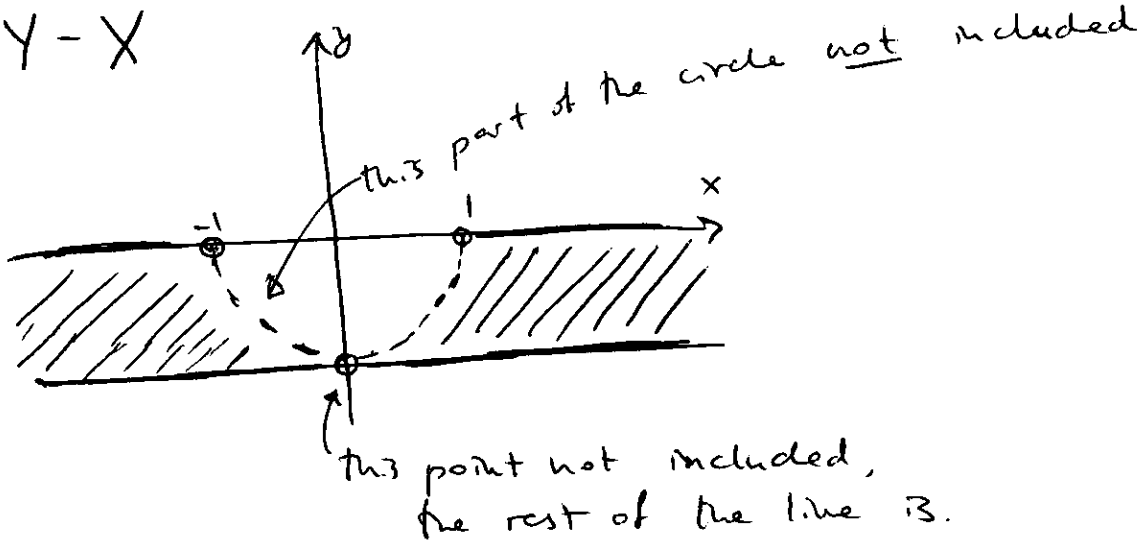






This part of the boundary is NOT included!

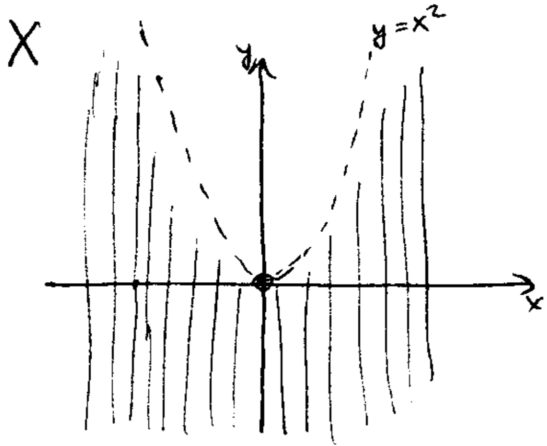
Y - X


y
x
-1
1
this part of the circle not included
this point not included,
the rest of the line is.

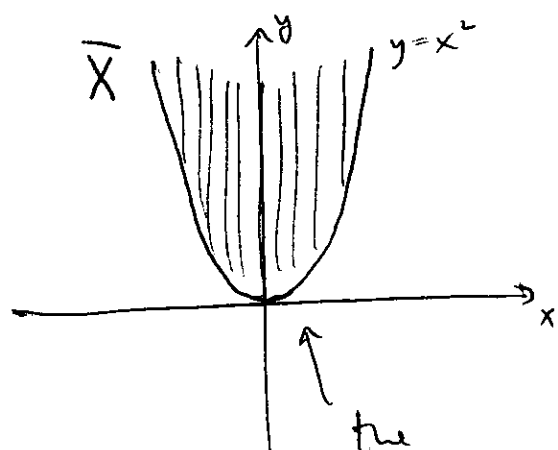
- **Section 1.6: Problem 2.** $A = \{0,2,4,6,8\}$, $B = \{1,3,5,7\}$. We get:
  (a) $\overline{A} = \{1,3,5,7\} = B$.
  (b) $\overline{B} = \{0,2,4,6,8\} = A$.
  (c) $A \cap \overline{A} = \varnothing$ (this is always true)
  (d) $A \cup \overline{A} = U$ (this is also always true)
  (e) $A - \overline{A} = A$ (also always true because $\overline{A}$ has no common elements with A)
  (f) $\overline{A \cup B} = \varnothing$: in our case, $A \cup B$ is the whole set U, so there's nothing in its complement.
  (g) $\overline{A} \cap \overline{B} = B \cap A = \varnothing$ (here the first equality is because of Parts (a) and (b); this is of course NOT always true).
  (h) $\overline{A \cap B} = U$: here $A \cap B$ is empty, therefore its complement is the whole universal set.
  (i)

$$\overline{A} \times B = \{(1,1),(1,3),(1,5),(1,7),(3,1),(3,3),(3,5),(3,7),\\ (5,1),(5,3),(5,5),(5,7),(7,1),(7,3),(7,5),(7,7)\}.$$

1.6 Problem 6 :

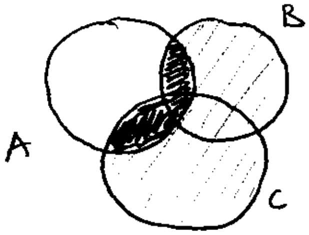


the set $\{(x,y) \in \mathbb{R}^2 : y < x^2\}$.
The parabola not included.



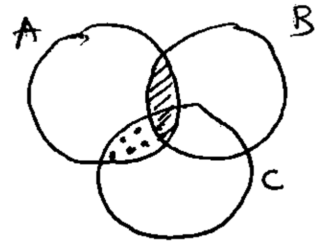
the parabola is included.

1.7 Problem 6



light shading: $B \cup C$

dark shading: $A \cap (B \cup C)$



shading: $A \cap B$

dots: $A \cap C$

shaded or dots:

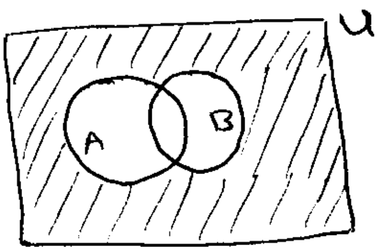
$(A \cap B) \cup (A \cap C)$

The drawings suggest:

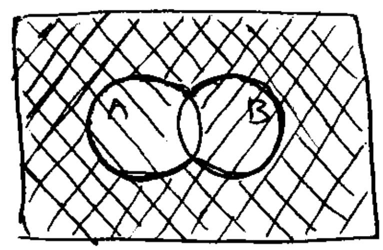
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

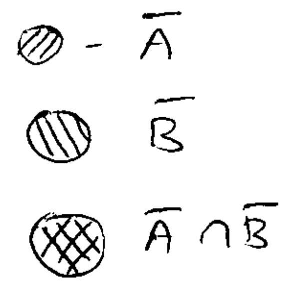
Can you prove it?

1.7 Problem 8



$\overline{A \cup B}$





we have: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Think of: why is this the same as De Morgan's law?

1.7 Problems 12, 14:

12: $(A - B) \cup (C \cap B)$

14: $(A - (B \cup C)) \cup A \cap B \cap C$

- **Section 1.8: Problem 2.**

  (a) By definition of the union of an indexed collection of sets,

$$\bigcup_{i=1}^{3} A_i = A_1 \cup A_2 \cup A_3.$$

  Note that in this problem, $A_3 \subset A_1$, so $A_1 \cup A_3 = A_1$ (the set A_3 does not have any elements that are not already in A_1). Then

$$A_1 \cup A_2 \cup A_3 = A_1 \cup A_2 = \{0,2,4,8,10,12,14,16,18,20,22,24,3,6,9,15,21\}.$$

  (b) The intersection of all three sets is:

$$\bigcap_{i=1}^{3} A_i = A_1 \cap A_2 \cap A_3.$$

  This is the set of elements that are common between all three sets. We get:

$$\bigcap_{i=1}^{3} A_i = A_1 \cap A_2 \cap A_3 = \{0,12,24\}.$$

- **Section 1.8. Problem 4.**

  (a)

$$\bigcup_{i \in \mathbb{N}} A_i = \{\ldots,-6,-4,-2,0,2,4,6\ldots\}.$$

  This is the set of all even integers.

  (b)

$$\bigcap_{i \in \mathbb{N}} A_i = \{0\}.$$

- **Section 1.8. Problem 8.** For each $\alpha \in \mathbb{R}$, the set $\{\alpha\} \times [0,1] = \{(\alpha, y) : y \in [0,1]$ is the vertical segment connecting the points $(\alpha, 0)$ and $(\alpha, 1)$ on the plane (see the picture). So we see that the union of all these segments is a horizontal strip:

$$\bigcup_{\alpha \in \mathbb{R}} \{\alpha\} \times [0,1] = \{(x,y) \in \mathbb{R}^2 : 0 \le y \le 1\},$$

  and their intersection is empty, since any two such segments do not have any common points.