1. The proofs below contain errors. Please identify these errors, and prove or disprove the original statement.

   (a) Statement: $\exists x \in \mathbf{R}$ such that for any $y \in \mathbf{R}, y \geq 0$, we have $xy = 2x$.

   Proof: Let $x \in \mathbf{R}$, and $y = 2$. Then, $y \geq 0$, and $xy = 2x$, as we wanted.

   (b) Statement: For all real numbers x, we have that $|x| > \sqrt{x^2 - 1}$.

   Proof. Let $x \in \mathbf{R}$. Then $0 > -1 \Longrightarrow x^2 > x^2 - 1 \Longrightarrow \sqrt{x^2} > \sqrt{x^2 - 1} \Longrightarrow |x| > \sqrt{x^2 - 1}$

2. Let $f : A \to B$ be a function. Let $C \subseteq A, D \subseteq B$.
   (a) Write down the definition of f is injective.
   (b) Write down the definition of f is surjective.
   (c) Write down the definition of $f(C)$
   (d) Write down the definition of $f^{-1}(D)$.

3. Prove that the function $g : \mathbf{R} - \left\{\frac{1}{2}\right\} \to \mathbf{R} - \{2\}$ defined by $g(x) = \frac{4x+2}{2x-1}$ is bijective.

4. Let A be a non-empty set and let $h : A \to A$ be a function. Prove that if $h \circ h$ is bijective then h is bijective.

5. Find an example of functions $f : A \to B$ and $g : B \to C$ so that f and $g \circ f$ are injective but g is not injective.

6. Find an example of functions $f : A \to B$ and $g : B \to C$ so that g and $g \circ f$ are surjective but f is not surjective.

7. Let $g : A \to C$ and $h : B \to C$ are functions so that h is bijective. Prove that there exists a function $f : A \to B$ so that $g = h \circ f$.

8. Define a relation on $\mathbf{N}$ by aRb iff $a^2 \equiv b^2 \pmod 5$. Show R is an equivalence relation. Show that $[2] = [3]$.

9. (a) Let $f : A \to B$ be a function and let $D \subseteq B$. Prove that

$$f^{-1}(B \setminus D) \subseteq A \setminus f^{-1}(D)$$

   (b) Let $f : A \to B$ be an injective function and let $C \subseteq A$. Prove that

$$f^{-1}(f(C)) \subseteq C$$

   Hence "=" holds as the other direction holds without assuming f is injective, as shown in class.

10. Prove that $1^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$ for all $n \in \mathbb{N}$.

11. Use induction to prove that $81 \Big| (10^{n+1} - 9n - 10)$ for every non-negative integer n.

12. Prove $(5)^{1/3}$ is irrational.