Questions from class

what is the curvature at of a path at the point where it reverses itself?

It is undefined, and the limit of the curvature as one approaches that point depends on the sense in which you take the limit.

Consider the upper half of the circle parameterized by

\[ r(t) = (\cos t^2, \sin t^2) \]

\[-\sqrt{\pi} < t < \sqrt{\pi} \]

Start at \((-1, 0) \rightarrow (1, 0) \rightarrow (-1, 0)\)

\[ \hat{\mathbf{x}} \]
clearly the limit of the curvature as one approaches \((1,0)\) from the left is 1 (remember curvature is indep. of parameterization) 

However, if the path is taken as a limit of the path

![Diagram](image)

as \(\varepsilon \to 0^+\), then the curvature at \((1,0)\) would approach \(\infty\) as \(\varepsilon \to 0^+\).

Similarly, a line segment has 0 curvature, so the curvature near the endpoints as one approaches would be 0.

However, if one thinks of this path as the
\[ \lim \]

Limit of the path on an ellipse

as \( a \to 0 \), then the curvature at the endpoints approaches \( \infty \) in this limit.