(1) The upper half of an ellipse defined by
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1; \quad y > 0, \]
has density \( \rho(x, y) = 2x^2y \). Denote this region \( D \).

(a) Find its mass by computing a double integral over \( D \).
(b) Find its mass by computing the line integral
\[ \oint_{\partial D} \vec{F} \cdot d\vec{r} \]
for an appropriately chosen vector function \( \vec{F} \).

(2) Consider the surface \( S \) defined as the side of the cylinder \( x^2 + y^2 = 1 \) that lies above the plane \( z = 0 \) and below the plane \( x + 2y + 3z = 12 \).

(a) Find the surface area of \( S \) by computing a double integral.
(b) Consider the integral
\[ I = \iint_S \nabla \times \vec{F} \cdot \hat{n} \, dS, \]
where \( \hat{n} \) denotes the outwards oriented unit normal (away from origin) of the surface \( S \). Find a vector \( \vec{G} = \nabla \times \vec{F} \) such that \( I \) yields the area of \( S \). Explain your reasoning. Hint: \( \nabla \cdot \nabla \times \vec{F} = 0 \).
(c) Verify that if we use \( \vec{F} = (zy, -xz, 0) \) in \( I \), then \( I = \text{Area}(S) \).
(d) Use Stokes’ theorem with \( \vec{F} = (zy, -xz, 0) \) in \( I \) to compute the area of \( S \).
(e) Let \( S_1 \) denote the part of the plane that lies inside the cylinder. Compute its area.
(f) Find a vector \( \vec{G}_1 = \nabla \times \vec{F}_1 \) such that
\[ I_1 = \iint_{S_1} \nabla \times \vec{F}_1 \cdot \hat{n}_1 \, dS \]
yields a multiple of the area of \( S_1 \). Here, \( \hat{n} \) is the upwards oriented unit normal of the surface \( S_1 \).
(g) For \( \vec{F}_1 = (2z, 3x - z, 0) \) in \( I_1 \), compute the area of \( S_1 \) using Stokes’ theorem.

(3) Consider the vector field \( \vec{F} = (y^3 + 3x^2y, 3x, z^2) \) and the curve \( C \) that is the intersection between the surface \( z = g(x, y) \) and the cylinder \( x^2 + y^2 = 1 \). Compute the work \( W \) done along \( C \) by \( \vec{F} \),
\[ W = \oint_C \vec{F} \cdot d\vec{r} \]
\( (C \) is oriented clockwise when viewed from above). Does \( W \) depend on the choice of \( g(x, y) \)?