(c) Recall that for a least squares fitting of data points \((p_1, w_1), \ldots, (p_n, w_n)\) with a quadratic function \(w(p) = ap^2 + bp + c\), we have the system
\[ Au = w + \varepsilon; \quad A = \begin{pmatrix} p_1^2 & p_1 & 1 \\ \vdots & \vdots & \vdots \\ p_n^2 & p_n & 1 \end{pmatrix}, \quad u = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \quad w = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}, \]  

(1)

where the \( j \)-th element of \( \varepsilon \) is the error \( w(p_j) - w_j \). The normal equations (see notes) are then

\[ A^T A u = A^T w, \]  

(2)

which essentially states that \( A^T \varepsilon = 0 \) (see this for yourself by multiplying the left- and right-hand sides of (1) by \( A^T \) and comparing (2)). Therefore, \( \varepsilon \) is orthogonal to the column space of \( A \). Since \( e = (1, 1, \ldots, 1)^T \) is in the column space of \( A \), we must have that \( e^T \varepsilon = 0 \). Therefore, the sum of the errors is zero. MATLAB will give a value of \( O(10^{-16}) \).

(d) The maximum of \( w(p) \) located at \( p \) where \( 2ap + b = 0 \), so \( p = -b/(2a) \). The coefficients \( a \) and \( b \) are computed from the least squares fit. You should have a value near \( p \approx 0.58 \) or 0.59.