1. Let \( a \) be the adjacent side of the angle, and \( c \) be the hypotenuse. By the bisector theorem, we have \( \frac{a}{7} = \frac{c}{25} \) (note it cannot be \( \frac{a}{25} = \frac{c}{7} \) since \( a < c \)). By Pythagoras’ theorem, we have \( a^2 + 32^2 = c^2 \), so we can solve this system to obtain the value of \( a \):

\[
\begin{align*}
a^2 + 32^2 &= \frac{25^2a^2}{7^2} \\
32^2 &= \frac{625 - 49}{49}a^2 = \frac{576a^2}{49} \\
a &= \sqrt{\frac{32^2 \cdot 7^2}{576}} = \frac{32 \cdot 7}{24} = \frac{28}{3}
\end{align*}
\]

Therefore the area of the triangle is \( \frac{a \cdot 32}{2} = \frac{448}{3} \).

2. The slope of the diagonal is \( \frac{315}{405} = \frac{7}{9} \). So we need to start by counting intersections in a \( 7 \times 9 \) tiling. This can be count manually. Alternatively, we note that when the diagonal crosses an horizontal or vertical line it intersects a new tile (except at the top corner), therefore the number of intersections in a \( 7 \times 9 \) tiling is \( 7 + 9 - 1 = 15 \). Finally, along the diagonal of the \( 315 \times 405 \) tiling, there are \( 45 \times 9 \) subtilings. Therefore, in total, there are \( 15 \times 45 = 675 \) intersections with the tiles.

3. We start by analyzing the reminders of \( 2^n \) mod \( 7 \):

\[
\begin{array}{cccccccc}
2^0 & 2^1 & 2^2 & 2^3 & 2^4 & 2^5 & \ldots \\
1 & 2 & 4 & 1 & 2 & 4 & \ldots \\
\end{array}
\]

We observe that the pattern repeats itself with a periodicity of 3.

(a) Therefore, the numbers of the form \( n = 3k \) satisfy that \( 2^n - 1 \) are divisible by 7.

(b) Moreover, all the numbers of the form \( 2^n + 1 \) leave a remainder of 2, 3 or 5 when divided by 7, so they are never divisible by 7.

4. Suppose that \( a \) divides both \( x = 21n + 4 \) and \( y = 14n + 3 \). Then \( a \) divides their difference \( z = x - y = 7n + 1 \), and also divides \( y - 2z = 14n + 3 - 2(7n + 1) = 1 \). Hence \( a \) divides 1 and must be equal to \( \pm 1 \). This proves that \( x = 21n + 4 \) and \( y = 14n + 3 \) are always relatively prime, so the fraction \( \frac{21n + 4}{14n + 3} \) cannot be reduced any further for any \( n \).