1. The triangle is isosceles. Thus, the height will divide the 36-degree angle into two of 18-degree angles. Using the definition of sine and cosine, we get: $\cos(72) = \sin(18)$. Then, using the double-angle and half-angle formulas we can transform that equality into the following equation:

$$0 = 8x^4 - 8x^2 + 1 + x = (2x - 1)(4x^2 - 2x - 1)(1 + x) = 0,$$

where $x = \cos(36)$. We know that $\cos(36)$ cannot be $1/2$ or negative. Thus $\cos(36) = (1 + \sqrt{5})/4$.

2. The polygon has 108-degrees angles. Let’s focus on the low-left angle of the polygon (its base). That vertice has one angle to the polygon and to two triangles. Thus, the triangles have one angle of size $(180 - 108/2) = 36$. Thus, the triangles angles are $32 - 90 - 54$.

Now, let’s focus on the other vertice of the base of the polygon. Here, there is a small triangle with 2 angles of 72-degrees, hence isosceles, with base 1. From the previous problem, we can get $\cos(72) = (\sqrt{5} - 1)/4$. Thus, we can find the size of the other sides of the triangle $x$ using $\cos(72)$:

$$\frac{1}{2x} = \cos(72) = (\sqrt{5} - 1)/4 \implies x = \frac{\sqrt{5} + 1}{2}.$$

It follows that the adjacent side to the 36-degree angle of the triangles has length $1 + x = (\sqrt{5} + 3)/2$.

Let’s use the definition of cosine one more time to get the length of the hypotenuse:

$$h = \frac{\text{adjacent}}{\cos(36)} = \frac{\sqrt{5} + 3}{2} \cdot \frac{4}{1 + \sqrt{5}} = 2 \frac{\sqrt{5} + 3}{1 + \sqrt{5}} = 1 + \sqrt{5}.$$

3. Let $K$ and $L$ be the total student loan of Jack and Jill respectively. Since Jill took twice much money than Jack, we have $L = 2K$. Jill only used $3/5$ of her loan, thus the total unused money by Jill is $L(2/5) = 4K/5$. We know that Jack used $1/3$ of the loan every year. Thus, there is $2/3K$ unused loan by the end of the first year, and he uses one third of that, so there are $(2/3)^2K$ by the end of the second year. Similarly,
there is \((2/3)^5 K\) unused loan after five years. Thus, Jill have in unused student loans:

\[
\frac{4}{5} K = \frac{4}{5} 3^5 \cdot \frac{1}{2^5} (13250) = 80493.75
\]