1. Twenty-one girls and twenty-one boys took part in a mathematical competition. It turned out that each contestant solved at most six problems, and for each pair of a girl and a boy, there was at least one problem that was solved by both the girl and the boy. Show that there is a problem that was solved by at least three girls and at least three boys.

2. The Fibonacci numbers $u_0, u_1, u_2, u_3, \ldots$ are a sequence of integers defined recursively as: $u_0 = 0$, $u_1 = 1$, and $u_{n+2} = u_{n+1} + u_n$, for any $n$

   a) Show that $u_{n+m} = u_{m-1}u_n + u_m u_{n+1}$ and conclude that any number which is a factor of $u_n$ and $u_m$ is also a factor of $u_{n+m}$. Show that $u_{kn}$ is divisible by $u_n$ for any $n$ and $k > 0$.

   b) In the first 20 Fibonacci numbers, which $u_n$ are prime numbers? Using a) or otherwise, show that $u_p$ can only be prime when $p$ is prime, although the converse is not true.

3. Suppose we have a basket of $Z$ eggs. If eggs are removed 2, 3, 4, 5 and 6 at a time, then there remains 1, 2, 3, 4 and 5 eggs, respectively. If eggs are removed 7 at a time, then no eggs remain in the basket. What is the smallest number of eggs that could have been in the basket? (Kody’s problem)

   Show your work.

4. Find all solutions of the system of equations $x^2 + x^2 y^2 + x^2 y^4 = 525$ and $x + xy + xy^2 = 35$.

5. Alicia is looking through binoculars as a distant ship sails away. Her eyes are 10 metres above sea level and the tip of the ship’s smokestack is 40 metres above sea level. How far away is the tip of the smokestack at the instant that it disappears from Alicia’s view? Assume that the Earth is a sphere of radius 6400 kilometres.