1. We have four equations $x_i + x_j x_k x_l = 2$ (for i, j, k, l distinct from 1 to 4). Thus, $x_i \neq 0$ for all i.

   If we multiply by x_i, to get $x_i^2 + x_i x_j x_k x_l = 2x_i$ and substract two equations we get:

$$\begin{aligned} x_i^2 - x_j^2 &= 2x_i - 2x_j \\ (x_i - x_j)(x_i + x_j) - 2(x_i - x_j) &= 0 \\ (x_i - x_j)(x_i + x_j - 2) &= 0 \end{aligned}$$

   Hence, $x_i = x_j$ or $x_i + x_j = 2$.

   Now, we have four cases:

   (a) All equal. Thus, the equation becomes $x - x^3 = 2$, $\implies x^3 - x + 2 = (x-1)(x^2 + 2x + 2) = 0$. One real solution: $x = 1$.

   (b) Three are equal and one different, say $x_1 = x_2 = x_3 = x \neq x_4 = y$. The system becomes $x + x^2 y = 2$ and $y + x^3 = 2$. By our observation above, we also know that either $x = y$ or $x + y = 2$ Thus, $x + x^2(2 - x) = 2$ implies $(x-1)(x+1)(x-2) = 0$. Then: $x = 1 \implies y = 1$; $x = -1 \implies y = 2 - x = 3$, and satisfy $y + x^3 = 2$; $x = 2 \implies y = 2 - 2 = 0$ doesn't work. Thus, the possibilities are $x = y = 1$ and $x = -1, y = 3$.

   (c) Two pairs equal, say $x_1 = x_2 = x$ and $x_3 = x_4 = y$. This case will have the equations $x + x^2 y = 2$ and $y + xy^2 = 2$. As before, $x + x^2(2 - x) = 2$ implies $(x-1)(x+1)(x-2) = 0$. Again, case $x = 1$ works and $x = 2$ doesn't. But now $x = -1, y = 3$ doesn't satisfy $y + xy^2 = 2$. Thus, this case reduces to the first one.

   (d) Only two are equal, say $x_1 = x_2 \neq x_3 \neq x_4$. Then, the equations $x_1 + x_3 = 2$ and $x_1 + x_4 = 2$, imply $x_3 = x_4$ and we are back to the previous case.

   (e) All are different, then we have the system $x_i + x_j = 2$ for all distinct i, j. This system has a unique solution $x_i = 1$.

   In summary, there five solutions to the original equations, namely $x_i = 1$ for all i, and $x_i = 3$ and $x_j = x_k = x_l = -1$ for $i = 1, 2, 3, 4$ and $j, k, l \neq i$.

2. Let a be the number of kids on overcrowded buses and b the number on non-overcrowded buses. Let x be the number of overcrowded buses, and y the non-overcrowded ones. Let's assume that all of them have the same capacity and call it M. It is easy to check the extreme cases ($x = 0$, or $y = 0$) and the cases with $x + y = 0$ or $a + b = 0$ make no sense to the problem. Then $A = a/x$ is the average number of kids on overcrowded buses, and $B = b/y$ is the average number of kids on non-overcrowded ones. By definition of overcrowded we get that, $A > .5M \geq B$. Then

$$A > B \implies Ax + Ay > Ax + By \implies \frac{A(x+y)}{Ax+By} > 1$$
$$\implies \frac{a}{x(a+b)} > \frac{1}{x+y} \implies \frac{a}{a+b} > \frac{x}{x+y}$$

Therefore, the percentage of children on overcrowded buses is greater than the percentage of overcrowded buses.

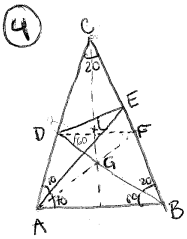
3. We have two cases:

   (a) If Aria sits at the beginning or at the end of the row. Then, we have two options for the person next to her (either D or E). For the next person, we have two options again (B or C, since E and D cannot sit together). Then, we have two options left in the next seat (the ones that have not taken sit yet) and finally the fifth person is fixed. Thus, we have $2 \times 2 \times 2 = 8$ ways to sit them if Aria sits at the beginning and 8 if Aria sits at the end.

   (b) If Aria sits somewhere in the middle of the row, then D and E must sit next to her, there are two ways of doing this (D to left or D to the right). All the evasion conditions are satisfied, so B and C can sit freely in the remaining two sits, this can be done in two ways. Since Aria has 3 choices for sitting in the middle, we conclude that there are $3 \times 2 \times 2 = 12$ ways of performing the arrangement in this case.

   Finally, we conclude that there are in total 12+16=28 ways of sitting with the required conditions.

(4)



0. Angle $\measuredangle ACB = 20$ by sum of angles of ΔABC.
1. Notice that ΔABC is isoceles
2. Draw DF such that $DF \parallel AB$, draw AF
3. Let G be the intersection of AF & DB
4. By $\parallel$ $\measuredangle ABD = \measuredangle BDF = 60$ and $\overline{AD} = \overline{BF}$
   Then ΔADF and ΔBDF are congruent
   Then $\measuredangle FAD = \measuredangle DBF = 20$
   $\Rightarrow \measuredangle BFA = 60 \Rightarrow \measuredangle AFD = 60$
   $\Rightarrow \measuredangle DGF = \measuredangle AGB = 60$ and ΔABG & ΔDGF are equilateral triangles.
5. Connect C and G. Since $\overline{DG} = \overline{GF}$, CG divides $\measuredangle DCF$ exactly in half. $\Rightarrow \measuredangle DCG = \measuredangle GCE = 10$

6. Since $\measuredangle GAC = \measuredangle ACE = 20$, $\measuredangle EAC = \measuredangle ACG = 10$, then $\Delta AGC \cong \Delta AEC$
   Then $\overline{AG} = \overline{CE}$
7. Notice that ΔAFC is isoceles, then $\overline{AF} = \overline{FE}$.
8. By 4, 6 & 7 we get $\overline{FE} = \overline{GF} = \overline{DF}$
   $\Rightarrow \measuredangle FDE = \measuredangle DEF$
9. Using ΔBDF sum of angles, we get $\measuredangle BFD = 180 - 80 = 100$
   $\Rightarrow \measuredangle DFE = 80 \Rightarrow \measuredangle DFE = \frac{100}{2} = 50$
10. Using ΔABE sum of angles, $\measuredangle AEB = 180 - 70 - 80 = 30$
11. By 9 & 10 $x = 50 - 30 = 20$