

1. First, let's count the numbers with the string 13. Consider three cases:
  - If the number is of the form  $ab13$ , then there are 20 options for  $ab$ , namely,  $01, \dots, 20$ . Note that 1313 is counted here.
  - If the number is of the form  $a13b$ , then there are 2 options for  $a$  and 10 options for  $b$ . Hence, there are  $2 \times 10 = 20$  numbers in this case.
  - If the number is of the form  $13ab$ , then there are 100 options for  $ab$ , namely,  $00, 01, \dots, 99$ . Note that 1313 is also counted here.

Therefore, there are  $20 + 20 + 100 - 1 = 139$  (minus 1 because 1313 is counted twice). Finally, there are 1919 numbers between 101 and 2019. Thus, the hotel has  $1919 - 139 = 1780$  rooms.

2. After the first pouring there are 2 oz of coffee in the first cup and 2 oz of coffee plus 4 oz of cream in the second cup, half of it is transfer back to the first cup, so now there are  $2 + 2/2 = 3$  oz of coffee plus  $4/2 = 2$  oz of cream in the first cup. Hence  $2/5$  of the liquid in the first cup is cream.
3. Since there are  $6 + 3 = 9$  black or gold marbles, the probability of drawing one of these colors is  $9/x$ , where  $x$  is the total number of marbles. Then we need  $3/7 = 9/x$ , so  $x = 21$ . Then the number of added white marbles is  $21 - (3 + 6 + 2 + 6) = 4$ .
4. Take it step by step. First, Alex spends all 75 blue tokens, and he gets  $75/3 = 25$  silver tokens and  $75 + 25 = 100$  red tokens. Then Alex exchange all 100 red tokens to get  $100/2 = 50$  silver tokens and 50 blue tokens. The following table summarizes the results:

	Red	Blue	Silver
	75	75	0
Exchange blues	100	0	25
Exchange reds	0	50	75
Exchange blues	16	2	91
Exchange reds	0	10	99
Exchange blues	3	1	102
Exchange reds	1	2	103

Therefore, Alex will have 103 silver tokens at the end.

**Another solution.** Let  $x$  and  $y$  be the amount of red and blue tokens (respectively) at a certain point. We will analyze the quantity  $S = 4x + 3y$  before and after the exchanges. If we exchange 2 red tokens we get 1 more blue, so the new quantity is  $S' = 4(x - 2) + 3(y + 1) = 4x + 3y - 5 = S - 5$ . If we exchange 3 blue tokens we get 1 more red, so the new quantity is  $S' = 4(x + 1) + 3(y - 3) = 4x + 3y - 5 = S - 5$ . Thus, regardless of the type of exchange the quantity  $S$  is reduced by 5 in each step. Finally note that the number of silver coins is the number of steps up to that point.

Initially the quantity is  $S = 4(75) + 3(75) = 525$  and reduces by 5 in each step. We stop when we reach the value  $S = 10 = 4(1) + 3(2)$ , since we cannot perform another exchange. Hence, the maximum possible number of exchanges that Alex can make is  $(525 - 10)/5 = 103$ . And this number corresponds to the number of silver coins he obtained.