1. Cantor proved that for any set S, the set P(S) of subsets of S has a larger cardinality than S.

[3](a) Let S = {question, this, sentence, no, has, two, threee, errors, erors} be a set of words. How many subsets does S have?

[7](b) Suppose you associate each element of S with a subset of S as follows:

"question" is associated with {question, sentence}
"this" is associated with {question, sentence, no}
"sentence" is associated with {"this"}
"no" is associated with {question, this, sentence, no, has}
"has" is associated with {}
"two" is associated with {"sentence", "this", "two", "threee"}
"threee" is associated with {"two", "this", "sentence"}
"errors" is associated with {"this", "sentence", "errors", "two"}
"erors" is associated with {"question", "sentence", "errors", "three"}

There are many subsets of S not associated with any element of S; however, what subset would you find using the construction that occurs in the proof of Cantor's theorem? In other words, find the mystery set.

a) S has 9 elements, so it has 2^9 subsets $\boxed{= 512 \text{ subsets}}$

b) M_f includes $\{x \mid x \in S$ and $x \notin f(x)\}$:

- ~~"question" ∈ f(question)~~
- "this" ∉ f(this)
- "sentence" ∉ f("sentence")
- ~~no ∈ f(no)~~
- "has" ∉ f(has)
- ~~two ∈ f(two)~~
- "three" ∉ f(three)
- ~~errors ∈ f(errors)~~
- "erors" ∉ f(erors)

(Mystery set) → $\boxed{M_f = \{\text{"this", "sentence", "has", "three", "erors"}\}}$

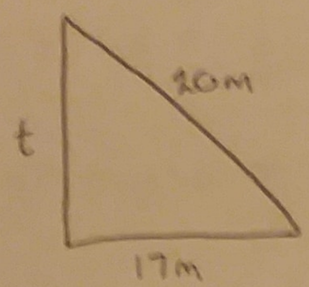
↳ set found using proof of Cantor's theorem

2. Pythagoras Word Problems

(a) Tony has got his kite stuck at the top of a very tall tree. He knows the string on his kite is 20 m long. When he pulls the string tight and holds the very end on the ground it touches 17 m from the bottom of the tree. If the ground is flat, how tall is the tree?

(b) Robert is using a 5m ladder to climb in his upstairs bedroom window. He finds that if he puts the base of the ladder 3.3m from the wall the top leans on the windowsill. How high from the ground is the windowsill?

a)



$t^2 + 17^2 = 20^2$

$t^2 = 20^2 - 17^2$

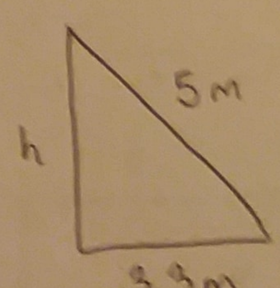
$t^2 = 400 - 289$

$\sqrt{t^2} = \sqrt{111}$

$t = 10.54m$

The tree is 10.54m tall.

b)



$h^2 + 3.3^2 = 5^2$

$h^2 = 5^2 - 3.3^2$

$h^2 = 25 - 10.89$

$\sqrt{h^2} = \sqrt{14.11}$

$h = 3.76\ m$

The windowsill is 3.76m from the ground.

3. (a) Solve the equation x = 1 + (1/x). (Hint: the solutions to the general quadratic equation $ax^2 + bx + c = 0$ are $x = (-b + (b^2 - 4ac)^{(1/2)})/2$ and $x = (-b - (b^2 - 4ac)^{(1/2)})/2$).

(b) Take a Golden Rectangle and remove the largest possible square so that you are left with a new smaller rectangle. Is this new rectangle a Golden Rectangle? If so, prove it. If not, show why not.

$x = 1 + \frac{1}{x}$

a) $x(x) = \left(1 + \frac{1}{x}\right)x$

$x^2 = x + \frac{x}{x}$

$(-x)x^2 = x + 1 \ (-x)$

$(-1)\ x^2 - x = 1 \ (-1)$

$x^2 - x - 1 = 0$

$(a = 1)$
$\rightarrow b = -1$
$c = -1$

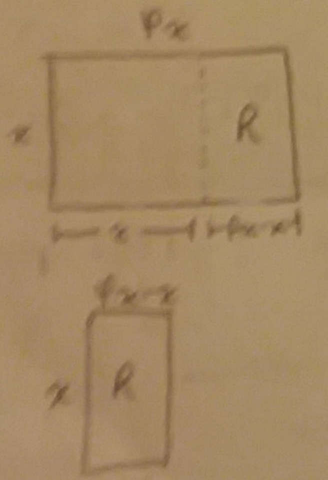
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-1)}}{2}$

$x = \frac{1 \pm \sqrt{1 - (-4)}}{2}$

$\boxed{x = \frac{1 \pm \sqrt{5}}{2}}$

b)



- dimensions of a golden rectangle are φx and x so it has a ratio of $\frac{\varphi x}{x}$
- after a square is removed, the remaining rectangle will have dimensions x and $\varphi x - x$
- the ratio of this rectangle (longer side / shorter side) is $\frac{x}{\varphi x - x} = \frac{x(1)}{x(\varphi - 1)} = \frac{1}{\varphi - 1}$
- $\frac{1}{\varphi - 1} = \varphi$, so the smaller rectangle is also a golden rectangle.

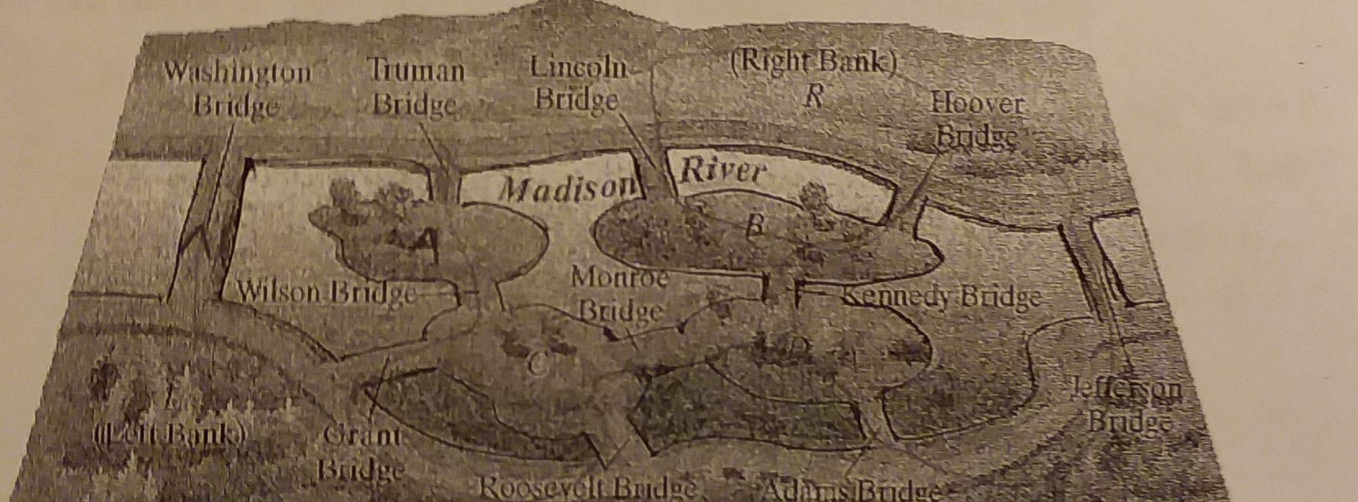
↳ $\varphi = \frac{1 + \sqrt{5}}{2} = 1.618$

↳ $\frac{1}{\varphi - 1} = 1.618$

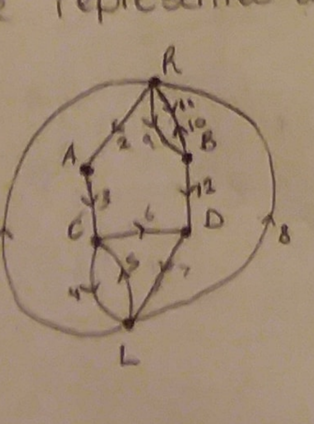
4. Solve the following problem(ignoring reference to example 5.3)

EXAMPLE 5.4 The Bridges of Madison County: Part 1

This is a more modern version of Example 5.3. Madison County is a quaint old place, famous for its quaint old bridges. A beautiful river runs through the county, and there are four islands (*A*, *B*, *C*, and *D*) and 11 bridges joining the islands to both banks of the river (*R* and *L*) and one another (Fig. 5-3). A famous photographer is hired to take pictures of each of the 11 bridges for a national magazine. The photographer needs to drive across each bridge once for the photo shoot. Moreover, since there is a $25 toll (the locals call it a "maintenance tax") every time an out-of-town visitor drives across a bridge, the photographer wants to minimize the total cost of his trip and to recross bridges only if it is absolutely necessary. What is the optimal (cheapest) route for him to follow?

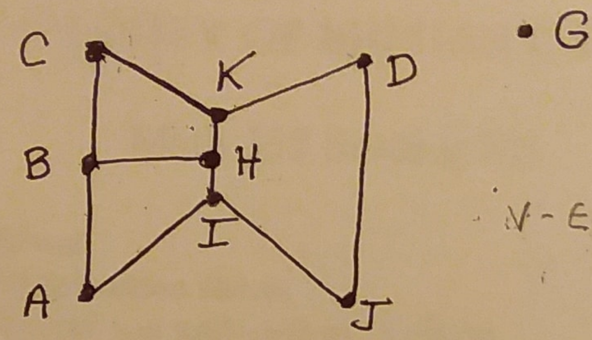


icture represented as an abstract graph:



- Since R, B, D, and L all have an odd degree, there is neither an Euler circuit or an Euler path
  - this means that one bridge will need to be crossed twice
- the cheapest route (represented in the graph on the left with numbers and directions) will cost $300 and is as follows:
- starting on Left Bank: Washington Bridge, Truman Bridge, Wilson Bridge, Grant Bridge, Roosevelt Bridge, Monroe Bridge, Adams Bridge, Jefferson Bridge, Lincoln Bridge, Hoover Bridge, Hoover Bridge, Kennedy Bridge

5. (a) Using the solid lines of the graph below, compute the Euler Characteristic V-E+F

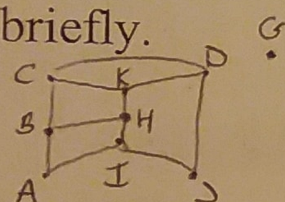


V = 9
E = 10
F = 4

$V - E + F = 9 - 10 + 4 \boxed{= 3}$

(The outside region counts as a region.)

(5) (b) If you added an edge from point C to point D, how would the number you just calculated change? Explain very briefly.

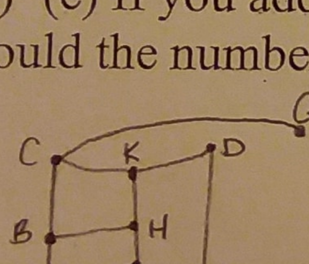


V = 9
E = 11
F = 5

$V - E + F = 9 - 11 + 5 \boxed{= 3}$

The Euler characteristic remains the same, since both an edge (-1) and face (+1) were added. The graph still has 2 parts, so the Euler characteristic is still 3.

(5) (c) If you added an edge from point C to point G, how would the number you calculated change? Explain.



V = 9
E = 11
F = 4

$V - E + F = 9 - 11 + 4 \boxed{= 2}$

The Euler Characteristic decreases to 2, since an edge was added (-1) but the number of faces and vertexes remained the same. This extra edge made the graph a connected graph, and the Euler Characteristic of connected graphs is always 2.