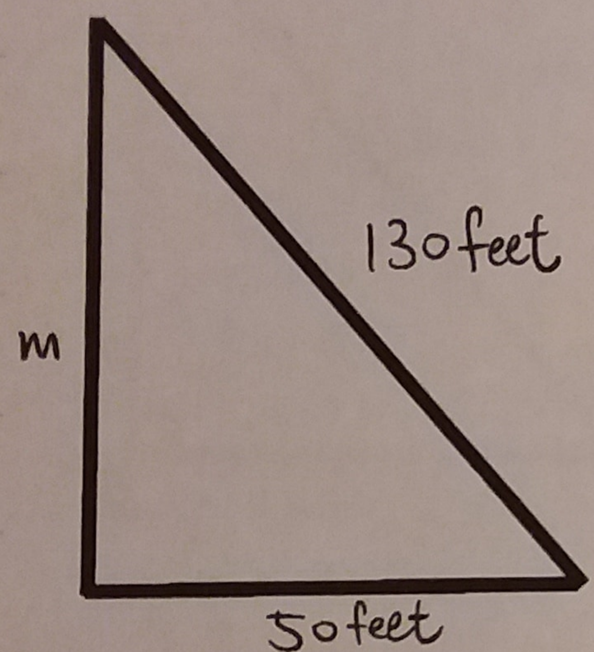
10. **Sand masting (H).** The sailboat named Sand Bug has a tall mast. The back-stay (the heavy steel cable that attaches the top of the mast to the back, or stern, of the sailboat) is made of 130 feet of cable. The base of the mast is located 50 feet from the stern of the boat. How tall is the mast?



By Pythagoras Theorem,

$$m^2 + (50)^2 = (130)^2$$

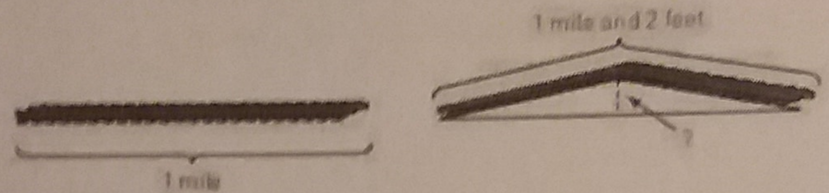
$$m = \sqrt{16900 - 2500}$$

$$m = \sqrt{14400}$$

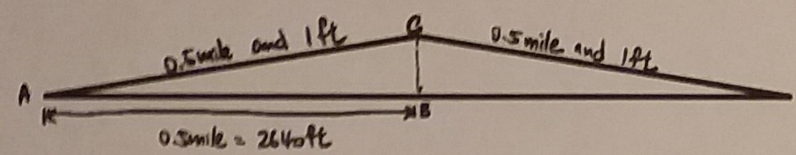
$$m = 120 \text{ feet.}$$

So, the mast is 120 feet tall.

15. Train trouble (II). Train tracks are made of metal. Consequently, they expand when it's warm and shrink when it's cold. When riding in a train, you hear the clickety-clack of the wheels going over small gaps left in the tracks to allow for this expansion. Suppose you were a beginner at laying railroad tracks and forgot to put in the gaps. Instead, you made a track 1 mile long that was firmly fixed at each end. On a hot day, suppose the track expanded by 2 feet and therefore buckled up in the middle, creating a triangle.



Roughly how high would the midpoint be? Now you may appreciate the click-clack of the railroad track.



Considering $\triangle ABC$,

$AB = 1 \text{ mile} \div 2 = 0.5 \text{ mile} = 2640 \text{ ft}$

$AC = (1 \text{ mile } \& \text{ } 2 \text{ feet}) \div 2 = 0.5 \text{ mile} + 1 \text{ ft} = 2641 \text{ ft}$

By Pythagoras Theorem,

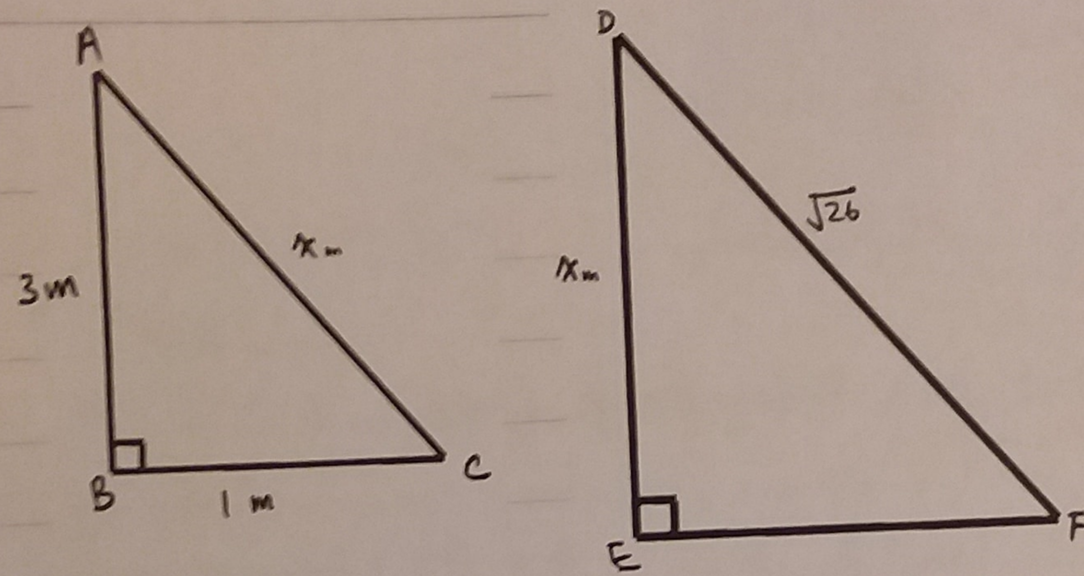
$AB^2 + BC^2 = AC^2$

$2640^2 + BC^2 = (2641)^2$

$BC = \sqrt{(2641)^2 - (2640)^2}$

$= 72.6705$ ft (correct to 4 decimal places)

28. **Ahoy there! (H)** Your exotic sailboat, which you named the *Pythagoras*, has two sails. Each one is the shape of a right triangle. The smaller sail has legs of length 1 and 3 meters and hypotenuse of length x meters. The larger sail has one leg whose length equals the length the hypotenuse of the shorter sail. The larger sail has a hypotenuse of length $\sqrt{26}$ meters. Find the lengths of all the sides of both triangular sails.



In $\triangle$ ABC, by Pythagoras Theorem

$$AB^2 + BC^2 = AC^2$$

$$3^2 + (1)^2 = AC^2$$

$$AC = \sqrt{10} = 3.1623 \text{ m (correct to 4 decimal places)}$$

In $\triangle$ ABC and $\triangle$ DEF,

$$AC = DE = \sqrt{10} \text{ meters}$$

In $\triangle$ DEF, By Pythagoras Theorem,

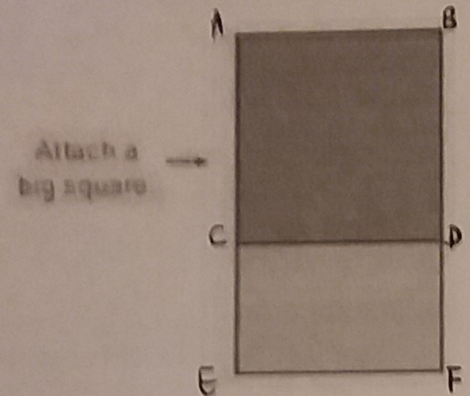
$$DE^2 + EF^2 = DF^2$$

$$(\sqrt{10})^2 + EF^2 = (\sqrt{26})^2$$

$$EF = \sqrt{26+10} = \sqrt{36} = 6 \text{ meters}$$

Thus, AC = $\sqrt{10}$ meters, DE = $\sqrt{10}$ meters, EF = 6 meters.

13. **Growing gold (II).** Take a Golden Rectangle and attach a square to the longer side so that you create a new larger rectangle. Is this new rectangle a Golden Rectangle? What if we repeat this process with the new, large rectangle?



If CDEF is a Golden Rectangle,

$$\frac{EF}{CE} = \frac{1}{\varphi - 1}$$

$EF = CD$

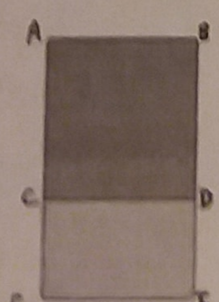
$\therefore CD = AC = AB$ (property of square)

$$\frac{AB}{AE} = \frac{1}{AC + CE} = \frac{1}{(1)+(\varphi-1)} = \frac{1}{\varphi}$$

$$\frac{AE}{AB} = \frac{\varphi}{1} = \varphi$$

Thus, $\frac{AE}{AB}$ equals to φ, rectangle ABCF is a golden rectangle.

16. **Comparing areas (ExH).** Let G be a Golden Rectangle having base b and height h, and let G' be the smaller Golden Rectangle made by removing the largest square possible from G. Compute the ratio of the area of G to the area of G'. That is, compute Area(G)/Area(G'). Does your answer really depend on b and h (the original size of G)? Are you surprised by your answer?



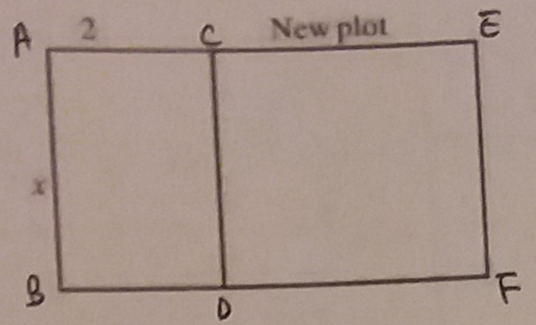
Let ABEF be G, let CDEF be G',

In G, $AE/AB = \varphi/1$, Area of G $= AE \times AB = \varphi$

In G', $CD/CE = 1/\varphi-1$, Area of G' $= CD \times CE = 1 \times (\varphi - 1) = \varphi - 1$

$$\text{Area}(G)/\text{Area}(G') = \frac{\varphi}{\varphi-1} = \frac{AE}{CE}$$

30. Adding a square. Your school's Healthy Eating garden plot is a rectangle measuring 2 meters by x meters. You decide to expand the area for more organic vegetables by adding a large square plot against the side measuring x meters. Write an expression in terms of x that gives the total area of the expanded garden. If the total area is 24 square meters, find x.



$AE \times AB = 24\ m^2$

In ABCD,

$AB = CD = x$ (property of rectangle)

In CDEF,

$CE = CD = DF = EF = x$ (property of square)

So that in ABEF,

$AE = AC + CE$

$AE = 2 + x$

$AE \times AB = \text{Area of } ABEF = 24\ m^2$

$(2+x)(x) = 24$

$x^2 + 2x = 24$

$0 = x^2 + 2x - 24$

By $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$,

$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-24)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{100}}{2}$

$x = 4$ or -6 (rejected, as any side of a rectangle must be > 0)

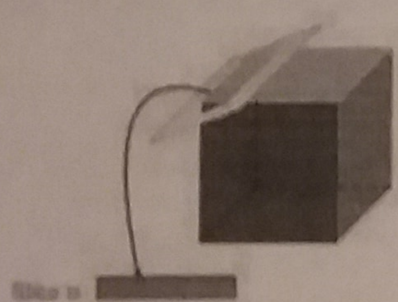
$\therefore x = 4$ meters

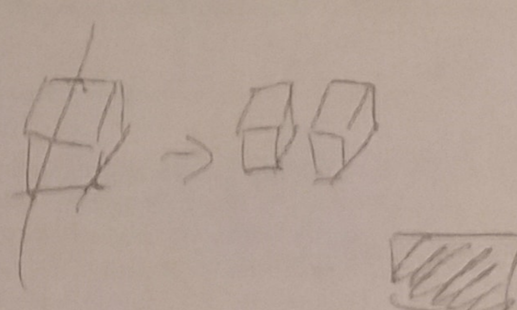
15. **Count.** For each of the regular solids, take the number of vertices, subtract the number of edges, add the number of faces. For each regular solid, what do you get?

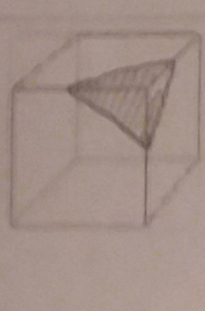
Solid Type	Vertices (V)	Edges (E)	Faces (F)	V - E + F
Tetrahedron	4	6	4	2
Cube	8	12	6	2
Octahedron	6	12	8	2
Dodecahedron	20	30	12	2
Icosahedron	12	30	20	2

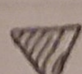
(Vertices - Edges + Faces) always equals to 2 in any regular solid.

16. **Cube slices (II).** Consider slicing the cube with a plane. What are all the different-shaped slices we can get? One slice, for example, could be rectangular. What other shaped slices can we get? Sketch both the shape of the slice and show how it is a slice of the cube.

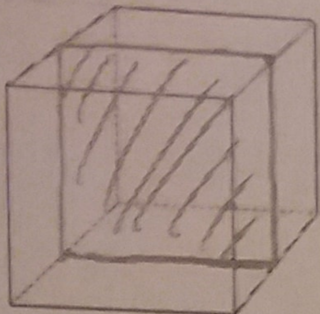




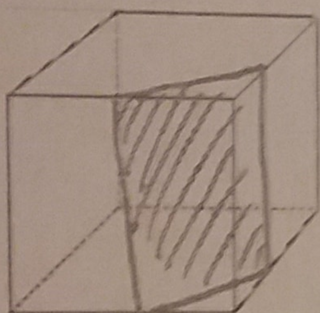




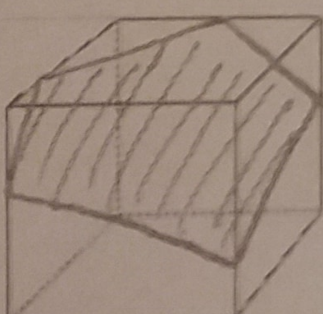
, a trianglar slice



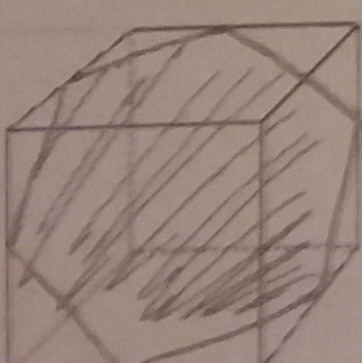
, a square

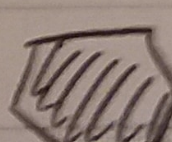


, a rectangular slice



, a pentagonal slice.





, a hexagonal slice