Math 335
Assignment 3

Section 2.4 - 2.7

2.4 - Crazy Clocks and Checking out Bars

3. a) $16 \bmod 7 \; \boxed{\equiv 2}$ $\quad (7\times 2)+2$
   b) $24 \bmod 7 \; \boxed{\equiv 3}$ $\quad (7\times 3)+3$
   c) $16 \times 24 \bmod 7$
   $\equiv 16 \times 3$
   $\equiv (14+2)\times 3$
   $\equiv 2 \times 3$
   $\boxed{\equiv 6}$
   d) $2\times 3 \; \boxed{\equiv 6}$
   e) The last two quantities are the same, which means that it doesn't matter when numbers are converted into modular equivalents.

7. a) $3724 \bmod 7 \equiv 0$, so it will be Saturday again.
   $\cancel{3500} + 200 + 24$
   $200 + \cancel{21} + 3$
   $\cancel{140} + 60 + 3$
   $\cancel{63} \equiv 0$
   b) $365 \bmod 7 \equiv 1$, so it will be Sunday.
   $\cancel{350} + 15$
   $\cancel{14} + 1$
   $\equiv 1$

12. * 0 1 6 9 0 0 0 0 3 0 3 4 is the correct UPC.
   Using $3d_1 + d_2 + 3d_3 + d_4 \ldots \equiv 0 \bmod 10$:
   → 0 24001 10691 3 =
   $3(0+4+0+1+6+1)+(2+0+1+0+9+3) = 51 \equiv 1 \bmod 10$ (incorrect)
   → 0 10010 20110 5
   $3(0+0+1+2+1+0)+(1+0+0+0+1+5) = 19 \equiv 9 \bmod 10$ (incorrect)

14. $3d_1 + d_2 + 3d_3 \ldots \equiv 0 \bmod 10$
   $3(0+8+0+1+0+0)+(2+5+0+1+7+\square) \equiv 0 \bmod 10$
   $42 + \square \equiv 0 \bmod 10$
   * missing digit is 8

2.6 - The Irrational Side of Numbers

12. Assume that $\sqrt{7}$ equals a rational number a/b, where a and b are reduced (don't have a common factor)

$$\sqrt{7} = \frac{a}{b} \rightarrow (\sqrt{7})^2 = \left(\frac{a}{b}\right)^2 \rightarrow 7 = \frac{a^2}{b^2} \rightarrow 7b^2 = a^2$$

↳ Therefore a^2 <u>and</u> a have 7 as a factor

If a has a factor of 7, we can rewrite it as $7n$

$$7b^2 = (7n)^2 = (7n)(7n) = 49n^2 \rightarrow \frac{7b^2}{7} = \frac{49n^2}{7} \rightarrow b^2 = 7n^2$$

↳ Therefore b^2 and b have 7 as a factor

This is a contradiction, since we originally assumed that a and b don't have any common factors, thus $\sqrt{7}$ must be an irrational number.

18. Assume E is a rational number a/b

$$13^{a/b} = 8 \rightarrow (13^{a/b})^b = 8^b \rightarrow \boxed{13^a = 8^b}$$

↳ Regardless of a, 13^a will always be odd, since an odd number multiplied by another odd number is always odd.

↳ Regardless of b, 8^b will always be even, since an even number multiplied by another even number is always even.

This is a contradiction, since an even number cannot equal an odd number, thus E must be an irrational number.

28. $\pi^2 = (\pi)(\pi)$

π must either be rational or irrational. Assuming that it is rational (a/b), where a and b represent whole numbers, $\pi^2 = \frac{a^2}{b^2} \cdot \frac{a^2}{b^2}$ must be rational, since any whole numbers remain rational when squared. The fact that π^2 is irrational is then a contraction, so π must also be irrational.

rational $\cdot$ rational = rational $\rightarrow$ π cannot be rational given that π^2 is irrational.

2.7 - Get Real

16.
$$\begin{array}{r} 1.433 \\ 15\overline{)21.500} \\ \underline{15} \\ 6.5 \\ \underline{6.0} \\ 50 \\ \underline{45} \\ 50 \\ \underline{45} \\ 5 \end{array}$$

remainder is the same, so pattern repeats

* $\frac{21.5}{15} = 1.4\overline{3}$

22. $5.63\overline{12} = W$

$$\begin{aligned} 10{,}000W &= 56312.\overline{12} \\ - \quad 100W &= 563.\overline{12} \\ \hline 9{,}900W &= 55{,}749.0 \end{aligned}$$

$$\frac{9{,}900W}{9{,}900} = \frac{55{,}749.0}{9{,}900}$$

* $W = 5.63\overline{12} = \frac{55{,}749^{\div 3}}{9{,}900^{\div 3}} = \boxed{\frac{18{,}583}{3{,}300}}$ prime

24. $71.23\overline{9} = W$

$$\begin{aligned} 1000W &= 71239.\overline{9} \\ -100W &= 7123.\overline{9} \\ \hline 900W &= 64{,}116 \end{aligned}$$

* $W = 71.23\overline{9} = \frac{64{,}116^{\div 6}}{900^{\div 6}} = \frac{10{,}686^{\div 2}}{150^{\div 2}} = \frac{5{,}343^{\div 3}}{75^{\div 3}} = \boxed{\frac{1{,}781}{25}}$ prime

OR

* $71.23\overline{9} = 71.24 = \frac{7124^{\div 4}}{100^{\div 4}} = \boxed{\frac{1{,}781}{25}}$ prime