2.4 - Crazy Clocks and Checking out Bars

3. a) \[16 \mod 7 = 2\]  \[(7 \times 2) + 2\]
   b) \[29 \mod 7 = 3\]  \[(7 \times 3) + 3\]
   c) \[16 \times 29 \mod 7\]
      \[= 16 \times 3\]
      \[= (14 + 2) \times 3\]
      \[= 2 \times 3\]
      \[= 6\]
   d) \[2 \times 3 \equiv 6\]
   e) The last two quantities are the same, which means that it doesn't matter when numbers are converted into modular equivalents.

7. a) \[3724 \mod 7 = 0\], so it will be Saturday again.
    \[
    \begin{align*}
    3724 + 200 + 24 \equiv 0 \\
    200 + 48 + 3 \\
    238 \equiv 60 + 3 \\
    63 \equiv 0 
    \end{align*}
    
    b) \[365 \mod 7 = 1\], so it will be Sunday.
    \[
    \begin{align*}
    365 + 15 \equiv 1 \\
    380 \equiv 1 
    \end{align*}
    
12. 0 1 6 9 0 0 0 0 3 0 3 4 is the correct UPC.
   Using \[3d_1 + d_2 + 3d_3 + d_4 \ldots \equiv 0 \mod 10:\]
   \[
   \begin{align*}
   & 0 24001 10691 3 = \\
   & 3(0 + 4 + 0 + 1 + 6 + 1) + (2 + 0 + 1 + 0 + 9 + 3) = 51 \equiv 1 \mod 10 \text{(incorrect)} \\
   & 0 10610 20110 5 = \\
   & 3(0 + 0 + 1 + 1 + 4) + (1 + 0 + 0 + 0 + 0 + 1 + 5) = 19 \equiv 9 \mod 10 \text{(incorrect)} 
   \end{align*}
   
13. \[3d_1 + d_2 + 3d_3 \equiv 0 \mod 10\]
   \[
   \begin{align*}
   & 3(0 + 8 + 0 + 1 + 0 + 0) + (2 + 3 + 0 + 1 + 7 + \square) = 0 \mod 10 \\
   & 42 + \square = 0 \mod 10 \\
   & \text{missing digit is 8}
   \end{align*}
   
14.
2.6 - The Irrational Side of Numbers

12. Assume that \( \sqrt{7} \) equals a rational number \( \frac{a}{b} \), where \( a \) and \( b \) are reduced (don't have a common factor).

\[
\sqrt{7} = \frac{a}{b} \Rightarrow (\sqrt{7})^2 = \left( \frac{a}{b} \right)^2 \Rightarrow 7 = \frac{a^2}{b^2} \Rightarrow 7b^2 = a^2
\]

Therefore \( a^2 \) and \( b \) have 7 as a factor.

If \( a \) has a factor of 7, we can rewrite it as \( 7n \):

\[
7b^2 = (7n)^2 = (7n)(7n) = 49n^2 \Rightarrow \frac{7b^2}{7} = 49n^2 \Rightarrow b^2 = 7n^2
\]

Therefore \( b^2 \) and \( b \) have 7 as a factor.

This is a contradiction, since we originally assumed that \( a \) and \( b \) don't have any common factors, thus \( \sqrt{7} \) must be an irrational number.

18. Assume \( E \) is a rational number \( \frac{a}{b} \).

\[
(\frac{a}{b})^2 = 8 \Rightarrow (\frac{a^2}{b^2})^2 = 8^2 \Rightarrow 13^2 = 8^2
\]

Regardless of \( a \), \( 13^2 \) will always be odd, since an odd number multiplied by another odd number is always odd.

Regardless of \( b \), \( 8^2 \) will always be even, since an even number multiplied by another even number is always even.

This is a contradiction, since an even number cannot equal an odd number, thus \( E \) must be an irrational number.

28. \( \pi^2 = (\pi)(\pi) \)

\( \pi \) must either be rational or irrational. Assuming that it is rational \( \left( \frac{a}{b} \right) \), where \( a \) and \( b \) represent whole numbers, \( \pi^2 = \frac{a^2}{b^2} \). \( \frac{a^2}{b^2} \) must be rational, since any whole numbers remain rational when squared; the fact that \( \pi^2 \) is irrational is then a contradiction, so \( \pi \) must also be irrational.

Rational • Rational = Rational \( \Rightarrow \pi \) cannot be rational, given that \( \pi^2 \) is irrational.
16. \[ \begin{array}{c}
15.3 \text{ over} 1,500 \\
\underline{15} \\
6.5 \\
6.0 \\
\underline{5.0} \\
4.5 \\
\underline{5.0} \\
4.5 \\
\underline{5} \\
\end{array} \]
remainder is the same, so pattern repeats

\[ \frac{21.5}{15} = 1.43 \]

22. \[ \sqrt{5.6312} = W \]

\[ \begin{array}{c}
10,000W = 56312.12 \\
- 100W = 563.12 \\
9,900W = 55,749.0 \\
9,900 \\
\end{array} \]

\[ W = 5.6312 = \frac{55,749}{9,900} = \frac{18,583}{3,200} \]

24. \[ \sqrt{71.239} = W \]

\[ \begin{array}{c}
1000W = 71239.9 \\
- 100W = 7123.9 \\
900W = 64,116 \\
\end{array} \]

\[ W = 71.239 = \frac{64,116}{900} = \frac{10,686}{150} = \frac{5343}{75} = \frac{1781}{25} \]

24. \[ \sqrt{71.239} = 71.24 \]

\[ \frac{7124}{100} = \frac{1781}{25} \]