Math 335
Assignment 2

Section 2.1 - 2.3

2.1 - Counting

8. $2^0 + 2^1 + 2^2 \dots + 2^{63}$

$\text{Sum} = 2^{63+1} - 1 = 2^{64} - 1 = 1.84 \times 10^{19}$ $\boxed{= 18,446,744,073,709,551,615}$

* The king had to give roughly 18,400,000,000,000,000,000 pieces of gold; he obviously did not realize how quickly exponential numbers grew. He had the minister of science executed either for tricking him or simply because he did not have enough gold.

17. All of the 3000's, + 300's (excluding 3300s), + 30's (excluding 330's), + 3's (excluding 33's).

3000s: $\frac{1}{10}$ $= 1000$

300s: $\frac{1}{10}\left(\frac{9}{10}\right)$ $= 900$

30s: $\frac{1}{10}\left(\frac{9}{10}\right)^2$ $= 810$

3s: $\frac{1}{10}\left(\frac{9}{10}\right)^3$ $= 729$

$\boxed{= \frac{3439}{10,000}}$

OR

$a\left(\frac{1-r^{n+1}}{1-r}\right)$ $a = \frac{1}{10}$ $r = \frac{9}{10}$ $n = 3$

$\frac{1}{10}\left(\frac{1-\left(\frac{9}{10}\right)^{3+1}}{1-\left(\frac{9}{10}\right)}\right) = \frac{1}{10}\left(\frac{1-\left(\frac{9}{10}\right)^4}{\frac{1}{10}}\right) = 1 - \left(\frac{9}{10}\right)^4 = 0.3439 = \frac{3439}{10,000}$

* 3439 of the first 10,000 natural numbers contain a 3.

A

2.2 - Numerical Patterns in Nature

6. *

Time in Months	Start	1	2	3	4	5	6	7
Number of Pairs	1	1	2	3	5	8	13	21

Month 0: 1 baby = 1 pair
Month 1: 1 adult = 1 pair
Month 2: 1 adult, 1 baby = 2 pairs
Month 3: 2 adults, 1 baby = 3 pairs
Month 4: 3 adults, 2 babies = 5 pairs
Month 5: 5 adults, 3 babies = 8 pairs
Month 6: 8 adults, 5 babies = 13 pairs
Month 7: 13 adults, 8 babies = 21 pairs

* Pattern: number of pairs follows the Fibonacci sequence
* To find the number of rabbit pairs in any month (F_n), add the previous number in the sequence (F_{n-1}) with the one before it (F_{n-2}). In general, $F_n = F_{n-1} + F_{n-2}$

16. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144

* 52 = 34 + 13 + 5
* 143 = 89 + 34 + 13 + 5 + 2
* 13 = 13 (cannot be written as the sum of more than one nonconsecutive Fibonacci number - it is a Fibonacci number itself)
* 88 = 55 + 21 + 8 + 3 + 1

22. 50 - 3 - 5 - 8 - 10 = 24 sticks remaining

As a sum of nonconsecutive Fibonacci numbers, 24 = 21 + 3

To win Fibonacci nim, always remove the smallest number of the sum of nonconsecutive Fibonacci numbers on your turn.

* In this example, remove 3 sticks next in order to win.

2.3 - Prime Cuts of Numbers

9. a) Yes : $8 \div 4 = 2$ and $45 \div 9 = 5$
b) Yes: $16 \div 4 = 4$ and $48 \div 8 = 6$
c) A nonprime divided by a nonprime will sometimes result in a prime. The complete factorization of a nonprime will always result in primes, but a single division may result in a nonprime as well, that would need to be factored further to reach a prime.

16. All the numbers are not prime: $111{,}111 \div 3 = 37{,}037$
If the number of 1's is a multiple of 3 or 2 the number will always be divisible by 3 or 11, which means it is not prime. There are an infinite number of multiples of 3, so there must also be an infinite amount of nonprime numbers composed only of 1s.

20. $n^2 + n + 17$

n = 1 : 19	n = 5 : 47	n = 9 : 107	n = 13 : 199
n = 2 : 23	n = 6 : 59	n = 10 : 127	n = 14 : 227
n = 3 : 29	n = 7 : 73	n = 11 : 149	n = 15 : 257
n = 4 : 37	n = 8 : 89	n = 12 : 173	$\boxed{n = 16 : 289}$

* The smallest value for n in which $n^2 + n + 17$ is not a prime number is $16 \rightarrow 16^2 + 16 + 17 = 289 \rightarrow 289 = 17 \times 17$.

25. $(91 \times 2) + 52 = 234$
234 is hypothetical original number
$234 + 103 = 337$
$7 \overline{)337}$ 48
$\underline{336}$ R1
1
$337 \div 7 = 48\ \boxed{R1}$
* The remainder is 1

26. a) No; the distance (on average) between primes gets further apart, even though it may sometimes work. $23 - 2 = 21 \rightarrow$ not a prime.
b) My guess is no, since the ratio of primes seems to always decrease. Even the best mathematicians, however, haven't answered this question. It is the same as the "Twin Prime Question," which remains unsolved.