1. For each of the following congruences, find the least nonnegative integer $x$ that satisfies it.

(a) \( \frac{60!}{31!} \equiv x \mod 31 \)  
(b) \( \frac{59!}{30!} = x \mod 31 \)

2. Let $p$ and $q$ be distinct odd prime numbers with $p-1 \mid q-1$. If $a \in \mathbb{Z}$ with $(a, pq) = 1$, prove that $q^{p-1} \equiv 1 \pmod{pq}$

3. Prove that 1729 is a Carmichael number.

4. Use Miller's test in base 2 to show that 1729 is composite.

5. If the following number represents a valid ISBN 10 code, then what is the missing digit?

   \[ y - 261 - 05073 - x \quad (\text{recall that } x = 10) \]