MATH 301 HW # 7

PROBLEM 1

(i) CALCULATE THE FOURIER TRANSFORM OF $1/(x^2+a^2)^2$ $\quad a>0$

(ii) CALCULATE THE INVERSE FOURIER TRANSFORM OF $1/(k^2+a^2)^2$ $\quad a>0$.

PROBLEM 2 USE THE FOURIER TRANSFORM TO SOLVE THE FREE SCHRODINGER EQUATION

$$iu_t + u_{xx} = 0 \qquad -\infty < x < \infty, \; t > 0$$

$$u(x,0) = f(x) \qquad \text{WITH} \quad f(x) \to 0 \text{ AS } |x| \to \infty.$$

PROBLEM 3 USE THE FOURIER TRANSFORM TO SOLVE THE DIFFUSION PROBLEM

$$u_t = D(u_{xx} + u_{yy}) \qquad -\infty < x < \infty, \; -\infty < y < \infty, \; t > 0$$

$$u(x,y,0) = f(x,y) \qquad f \to 0 \text{ AS } |x| \to \infty, \; |y| \to \infty$$

$$u \to 0 \text{ AS } |x| \to \infty, \; |y| \to \infty.$$

PROBLEM 4 FIND AN INTEGRAL REPRESENTATION FOR THE SOLUTION TO

$$u_t = D_0 u_{xx} + D_1 u_{xxxx} \qquad -\infty < x < \infty, \; t > 0 \qquad D_0, D_1 \text{ CONSTANTS}$$

$$u(x,0) = f(x) \qquad f \to 0 \text{ AS } |x| \to \infty,$$

BY FIRST FINDING THE DISPERSION RELATION.

THEN, FOR THE DISPERSION RELATION IN THE FORM

$$u = e^{ikx + \sigma t}$$

PLOT σ VERSUS k FOR EACH OF 4 CASES

(i) $D_0 < 0, \; D_1 < 0$ (ii) $D_0 > 0, \; D_1 > 0$ (iii) $D_0 < 0, \; D_1 > 0$

(iv) $D_0 > 0, \; D_1 < 0$.

WHAT DOES THIS TELL US PHYSICALLY?

PROBLEM 5 BY USING THE CONVOLUTION THEOREM FOR FOURIER TRANSFORMS DERIVE THE FOLLOWING IDENTITY

$$\int_{-\infty}^{\infty} [f(x)]^2 \, dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\hat{F}(k)|^2 \, dk.$$