MATH 301 HW 10

PROBLEM 1 CONSIDER $y' + y = f(t)$ WITH $f(t+1) = f(t)$ WITH INITIAL VALUE $y(0) = y_0$.

(i) SHOW HOW TO FIND y_0 SO THAT THE SOLUTION IS PERIODIC WITH $y(t+1) = y(t)$.

(ii) FOR THIS VALUE OF y_0 CALCULATE THE PERIODIC SOLUTION WHEN $f(t) = 1$ FOR $0 \le t \le 1/2$ AND $f(t) = 0$ FOR $1/2 \le t \le 1$ WITH $f(t+1) = f(t)$.

PROBLEM 2 CONSIDER THE FOLLOWING INITIAL VALUE PROBLEM FOR $y(t)$:

$$y'''' + k y''' + y'' + y' = e^{-t} \quad t \ge 0, \quad k \text{ REAL WITH } k \ge 0$$

$$y(0) = 0, \; y'(0) = 0, \; y''(0) = 1, \; y'''(0) = 4$$

(i) calculate the Laplace transform of $y(t)$, denoted by $Y(s)$, in the form $Y(s) = P(s)/Q(s)$ where P, Q polynomials.

(ii) IF $k = 2$ prove that $y(t)$ IS BOUNDED AS $t \to \infty$ AND calculate $\lim_{t\to\infty} y(t)$.

(iii) FOR what range of k is $y(t)$ BOUNDED AS $t \to \infty$?

(iv) FIND THE BEHAVIOR OF $y(t)$ AS $t \to \infty$ when $k = 1$. what happens to $y(t)$ when $0 < k < 1$?

PROBLEM 3 USING LAPLACE TRANSFORMS FIND THE SOLUTION TO THE DIFFUSION EQUATION

$$u_t = u_{xx} \quad 0 < x < \infty, \; t > 0$$

$$u(0,t) = 1, \; u(x,0) = e^{-x}, \; u \to 0 \text{ AS } x \to \infty \text{ FOR } t \text{ FIXED}$$

PROBLEM 4 FIND AN INTEGRAL REPRESENTATION FOR THE SOLUTION TO

$$u_{tt} = -u_{xxxx} \quad -\infty < x < \infty, \quad t > 0$$

WITH $u(x,0) = g(x)$ AND $u_t(x,0) = 0$

USING FOURIER TRANSFORMS. WE ASSUME $u, u_x, u_{xx}, u_{xxx} \to 0$ AS $|x| \to \infty$.