1. Introduction

1.1. Overview. [Short paragraph with one or two sentences.] The goal of this article is to study tradeoffs between the length of a final exam period the number of students with exam conflicts. We also study the effect of allowing large classes to give individual exams for each section.

1.2. Motivation. [Short paragraph with one or two sentences.] Our motivation is that students and teaching staff tend to prefer a short exam period, but exam conflicts creates additional work for the teaching staff.

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1.3. **Specific Questions in this Line of Research.** [One to three paragraphs.]

The first type of questions we study involve varying the length of the exam period, and seeing how the number of conflicts increases as the period shortens. Specifically we want to know under what circumstances there is a threshold phenomenon, meaning that the number of conflicts is extremely sensitive to the length of the exam period in a small interval of lengths, and is otherwise fairly insensitive to the length. We wish to study this question for a number of exam scenarios, based on the size of the university and versatility of the programs.

We will also examine if allowing large classes to schedule separate exams for each of its sections can significantly reduce the exam period length.

**2. Models**

Give some precise models you can work with. Start with a simple one; if there are more elaborate models, describe them.

2.1. **Basic Model.** The simplest model of exam schedule is the classical model of exam scheduling formulated as an ILP, akin to a graph colouring problem. The basic setup is that we have \( n \) classes and \( m \) possible exam periods, and we are given the set, \( P \), of pairs \((i, j)\) with \(1 \leq i, j \leq n\) such that classes \(i\) and \(j\) have common students. For each possible exam schedule, assigning classes to exam periods, we set for \(i = 1, \ldots, n\) and \(k = 1, \ldots, m\):

\[
x_{ik} = \begin{cases} 
1 & \text{if class } i \text{ is assigned colour } k, \text{ and} \\
0 & \text{otherwise.} 
\end{cases}
\]

Conversely, if \(x_{ik}\) are variables such that are either 0 or 1 and such that

\[
(1) \quad \sum_{k=1}^{m} x_{ik} = 1, \quad \text{for all } i = 1, \ldots, n
\]

then we can interpret the \(x_{ik}\) as determining an assignment of classes to exam periods. In this description, an exam schedule has no conflicts if for all \((i, j) \in P\) and all exam periods \(k = 1, \ldots, m\) we have

\[
(2) \quad x_{ik} + x_{jk} \leq 1 \quad \text{for all } (i, j) \in P \text{ and } k = 1, \ldots, m.
\]

So a conflict free schedule is equivalent to the feasibility of \(x_{ik}\) subject to (1) and (2).

2.2. **More Refined Modeling for Classes.** A fancier model is to replace (2) by introducing variables \(y_{ijk} = 0, 1\) subject to

\[
x_{ik} + x_{jk} \leq 1 + y_{ijk}.
\]

Then the minimum value of \(y_{ijk}\) is 0 if \(i, j\) aren’t both assigned the colour \(k\), but otherwise \(y_{ijk}\) must be 1; one can then seek to minimize

\[
\sum_{k} \sum_{(i,j) \in P} y_{ijk} w_{ij}
\]

where \(w_{ij} > 0\) is some measure of how bad it is to assign classes \(i\) and \(j\) to the same exam period.
2.3. **Modeling Students in Classes.** A refinement of the above model would be to keep track of individual students; we could try to minimize the number of students exam hardships, i.e., three exams in a 24 hour period, or get a more accurate count of how many conflicts there will be.

This can be modeled by the data that specifies for each of $s$ students, the subset of classes the student is taking. So for $\ell = 1, \ldots, s$ we are given the subset $S_\ell \subset \{1, \ldots, n\}$ of classes that the $\ell$-th student is taking.

Given this data, in terms of the variables $x_{ij}$ above, the $\ell$-th student does not have an exam hardship during any four consecutive exams (representing a 24-hour exam period) provided that for each $k = 1, \ldots, n-3$ and each $\ell = 1, \ldots, s$ we have

$$\sum_{j=k+1}^{k+3} \sum_{i \in S_\ell} x_{ij} \leq 2.$$  

Otherwise we could introduce a variable $h_\ell$, which is 1 if student $\ell$ has some exam hardship and 0 otherwise, and then require

$$\sum_{j=k}^{k+3} \sum_{i \in S_\ell} x_{ij} \leq 2 + 2h_\ell;$$

hence the minimum value of $h_\ell$ is 0 if student $\ell$ has no hardships, and 1 otherwise. Then we can use the sum of the $h_\ell$ in the objective function.

3. **Data**

Using the UBC website, we have gathered the course requirements for students majoring in the following degree programs: (1) B.A. in Mathematics, (2) B.Sc. in Computer Science, (3) B.A. in Psychology. We have gathered class sizes in the last year for the core courses in such programs, and taken a sample of possible electives. See Appendix A for this information.

Using the above data, we have created data sets of sample course selections for students in years 1—5 in each of these majors, by state how this is done.

We have also created some data sets of the number of exam conflicts in each pair of courses by state how this is done.

4. **Our Algorithms**

Describe the LP/ILP/QP software you are using. We use the Gurobi libraries in Python to solve the ILP’s based on the above ILP’s.

If you are using other algorithms, describe them.

5. **Our Main Current Results**

6. **Obstacles Encountered So Far**

7. **Questions for Future Research**

Our next steps are to describe your very next step or two.
Give a list of specific questions that would be good to study, including (1) some that you might have time for, and (2) one or two that you won’t have time for.

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