Markowitz Model, Non-linear (quadratic programming)

In preparation (Ch 24 Vanderbei)

Quadratic Programming:
maximize/minimize: \( U = U(x') = c^T x + \sum_{i,j} a_{ij} x_i x_j \)

\[ \text{subject to: } A x \leq b \]
maybe \( x \geq 0 \)
[maybe \( x \in \mathbb{Z}^n \) (integers)]

Look at \( \max/\min \ x = (x) \) one variable

Maximize \( U = x^2 \)

maximize \( U = -x^2 \)

\( U(x) \) unbounded \( \left\{ \begin{array}{c} x \text{ free, } x \in \mathbb{R} \\ \text{bounded} \end{array} \right. \)

\( s.t. \ y \leq -1 \leq x \leq 1 \)

Example (convex)
Maximin

Convex function:

\[ f(x) = x^2 \]

\[ f'(0) \text{ doesn't help} \]

Convex \[ \iff f'' > 0 \]

Worse with 2 variables:

\[ f(x^2) = f(x_1, x_2) = x_1^2 + x_2^2 = (\text{distance to (0,0)})^2 \]

\[ \text{say!} \]

\[ \text{perturb a little} \]

\[ \text{have to examine each corner} \]

Look at:

\[ -1 \leq x_1 \leq 1, \quad -1 \leq x_2 \leq 1, \quad \ldots, \quad -1 \leq x_n \leq 1 \]

Look at:

\[ f(x^2) = x_1^2 + x_2^2 + \ldots + x_n^2 \]

\[ \text{2n inequalities} \]

\[ \text{2^n vertices} \]

\[ f = 0 \]

Claim: If \( f(x^2) \) is concave down, locally must check every vertex.

\[ x \in \mathbb{R}^n \]

\[ \text{Convex, local properties can guarantee global maximum} \]