- Finish scheduling with wait times
- Start Markowitz model, quad programming
- Remarks on homework (due by Thursday)

**HW 2:** LP: $x_1, x_2$: max $\frac{1}{2}x^T x$ s.t. $A x \leq b$, $x \geq 0$

$A x \leq b$

- $x \in \mathbb{R}$

**find** $x_2 = 10.1$
- $x_1 = 3.7$ for relaxed LP

$solve$, optimum 108 (low)

- $x_2 \leq 10$
- $x_2 \geq 11$

$ LP$

- $x_2 = 9$, $x_1 = 6.8$
- $z = 200$

$solve$
- $x_1 = 2$

$feasible integer solution$

$x_1 = 2$, $x_2 = 10$

$x \geq 3$

$need some feasible col before you can eliminate any branch$

objective = 150

(See Sept 28 examples)
Please note:

(1) You may work together on homework, but you must write up your own solutions individually. In particular, you must write your own code, spreadsheets, etc.

(2) You must acknowledge with whom you worked (specify their gradescope.com email addresses). You must also acknowledge any sources you have used beyond the textbook and class material.

(3) When you submit your homework to gradescope.com, you need to put the solutions to different problems on different pages; gradescope.com will ask you to identify which pages correspond to which problems.

(1) Use branch and bound to solve the integer linear program max $\tilde{c}^T \tilde{x}$ subject to $A\tilde{x} \leq \tilde{b}$ and $\tilde{x} \geq 0$ and $\tilde{x} \in \mathbb{Z}^2$ (i.e., $x_1, x_2$ must be integers)

$$\tilde{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}, \quad \tilde{b} = \begin{bmatrix} 29.8 \\ 7.3 \end{bmatrix}$$

Do not make use of the specific properties of $A, \tilde{b}, \tilde{c}$ in this problem (i.e., that they all have non-negative entries/coefficients). Specifically:

(a) Enter the corresponding LP into your LP software; you should find that the optimum solution is $x_1 = 0.9, x_2 = 6.4$, and $z = 22.8$.

(b) Try the following branches: $x_2 \leq 6$ and $x_2 \geq 7$. If you need to explore the $x_2 \leq 6$ branch further, divide this branch into $x_2 \leq 5$ and $x_2 = 6$; if you need to explore the $x_2 \geq 7$ branch further, divide this branch into $x_2 = 7$ and $x_2 \geq 8$. (You should find that the branch $x_2 \geq 8$ is infeasible.)

(c) When you reach a branch with $x_2$ fixed, branch on $x_1$ in a similar fashion (solve the relaxed LP, and round up and down).

(d) Complete the branch and bound, and make a diagram of the result.

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- Soon start Markowitz model, quad prog.

(Refs: Vanderbei, Ch 24; old handout of mine)

Before that: scheduling!

\[
\min \ x_n - x_1 \quad \text{s.t.,} \\
\sum_{i,j} W(i,j) \leq x_j \quad (i,j) \in E \\
x_i's \ are \ real, \ unbounded
\]

Wait times might be: 1, 20, 3, 3, 21, 5, 6, 3, 7, 17, ... - not necessarily integers

Duel:

\[
\sum_{i,j} Y_{ij} \left( x_i + W(i,j) \right) \leq x_j
\]

\[
\sum_{i,j} W(i,j) \leq \sum_{i,j} Y_{ij} (x_j - x_i)
\]

So duel:

\[
\sum_{i,j} Y_{ij} W(i,j) \quad \text{is a lower bound on} \quad x_n - x_1
\]

If \( \sum_{i,j} Y_{ij} (x_j - x_i) \) gives \( x_n - x_1 \)

so take \( x_k \) such:

\[
\sum_{i} Y_{ik} - \sum_{i} Y_{ki} = \begin{cases} 
1 & \text{if } k = n \\
-1 & \text{if } k = 1 \\
0 & \text{if } k = 2, \ldots, n-1
\end{cases}
\]
Example:

\[ y_{13} = \begin{cases} 1 \quad (x_1 + 20 \leq x_3) \\ 0 \quad \text{otherwise} \end{cases} \]

\[ y_{3n} = \begin{cases} 1 \quad (x_3 + 30 \leq x_n) \\ 0 \quad \text{otherwise} \end{cases} \]

\[ x_1 + x_3 + 50 \leq x_3 + x_n \]

\[ x_1 + 50 \leq x_n \]

\[ 50 \leq x_n - x_1 \]

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Must have integer solution by Theorem (total unimod) [sum]

\[ y_{i1} \geq 0 \quad \text{so} \quad y_{1n} + \cdots + y_{n-1,n} = 1 \]

If \[ y_{i1} \geq 0 \]

30 integers