Groups on Friday; if you can't make there will be a piazza.com page

Last time:
Class of ILP: "graph colouring"

\[ G = (V, E) \]
weight : \( E \rightarrow \mathbb{R}_{\geq 0} \)
\# colours

\[
\text{minimize } \sum \text{ weights of edges that are monochromatic}
\]

\[ \begin{array}{c}
\text{n vertices } v_1, \ldots, v_n \\
\text{k colours } \{1, \ldots, k\} \\
\text{"colouring" map } V \rightarrow \{1, \ldots, k\}
\end{array} \]
edges with endpoints of same colour: take \( \sum \) weights.

\[
\{x_{ij}\} \quad i = 1, \ldots, n \\
\quad j = 1, \ldots, k
\]
\[ \begin{cases} 
1 \text{ if vertex } i \text{ has colour } j \\
0 \text{ otherwise}
\end{cases} \]

For \( i_1, i_2 = 1, \ldots, n \) (\( i_1 \neq i_2 \), \( i_1 < i_2 \)) have weight \( w_{i_1i_2} \)
(Problem: Minimize \( \sum_{j=1}^{k} \sum_{i_1, i_2} w_{i_1, i_2} \))

Each color occurs once:

\( x_{11} + x_{12} + \ldots + x_{1k} = 1, \quad x_{ij} \in \{0, 1\} \)

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**Structure:**

LP - standard \( \equiv \) Application

ILP - rest: Bipartite Matching \( \times \) Apps

Bin Packing \( \equiv \) Apps

Vertex Cover \( \equiv \) Apps

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Graph Colouring \( \equiv \) App: Exam Scheduling

Bin Packing:

- Want to pack things into bins

Bin Packing: Rolls of Wrapping Paper

- Roll 1: 30 m, Cost $10
- Roll 2: 50 m, Cost $15

Gifts: gift 1: 5 m, gift 2: 1 m, \ldots, gift n 2 m

- Want to pack gifts into rolls at min cost.
e.g., \( g_1, \ldots, g_n \) reals

bins 1, \ldots, 3 30 m capacity
bins 4, \ldots, 7 50 m capacity

Can I pack \( g_1, \ldots, g_n \) into bins? Feasible

\( g_1 = 28, g_2 = 3, \ldots \)

Gifts \( \{1, \ldots, n\} \) \( \rightarrow \) Bin 1, \ldots, 3, 4, \ldots, 7

Let \( x_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ gift goes into bin } j \\ 0 & \text{otherwise} \end{cases} \)

Constraints: For each \( i \),

\[ x_{i1} + x_{i2} + \ldots + x_{i7} = 1 \]

Bin 1:

\[ x_{i1} g_1 + x_{i2} g_2 + \ldots + x_{i7} g_n \leq 30 \text{ m} \]

Bin 2:

\[ x_{i2} g_2 + \ldots + x_{i7} g_n = \sum_{i=1}^{n} x_{i2} g_i \leq 30 \text{ m} \]

Bin 3:

\[ \sum_{i=1}^{n} x_{i3} g_i \leq 30 \text{ m} \]

Bin 4:

\[ \sum_{i=1}^{n} x_{i4} g_i \leq 50, \text{ etc.} \]

If some gifts don't fit anywhere

\[ x_{i1} + x_{i2} + \ldots + x_{i7} \leq 1 \]

Maximize

\[ \sum_{i,j} x_{ij} \cdot \text{(value of gift } i) \]