- Branch & Bound (with Gurobi & without)
- More applications.

I'll illustrate some bad ideas in Branch & Bound.

Don't do the following:

\[
\begin{align*}
\max & \quad x_1 + 1000 x_2 \\
\text{s.t.} & \quad x_1 + 100 x_2 \leq 750.5 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

1. What happens if we restrict \( x_1, x_2 \) to be integers?

Roughly speaking: this very simple but useful technique for learning about Branch & Bound.
Step 1: Solve LP! So $x_1, x_2$ don't have to be integers! We get:

$$x_2 = \frac{750.5}{100}, \quad x_1 = 0$$

**LP:**

$$x_1 = 0, \quad x_2 = 7.505, \quad \text{Obj} = 7505$$

**LP and add $x_2 \leq 6$**

$$x_2 = 6, \quad x_1 = 150.5 \quad \text{Obj} = 6150.5$$

Eliminated

**LP and add $x_2 = 7$**

$$x_2 = 7, \quad x_1 = 50.5 \quad \text{Obj} = 7050.5$$

Eliminated

**LP and add $x_2 \geq 8$**

No solution

$x_1, x_2$ integers

Want to eliminate

Infeasible!

Infeasible!

Feasible

Integral!!!

Integer Solution

Feasible

Integer Solution

Eliminated
This doesn't work well:

\[ \text{LP: } x_1 = 0 \]
\[ x_2 = 7.505 \]
\[ \text{Obj: } 7505 \]

\[ x_2 \leq 99 \]
\[ \text{feasible} \]

\[ x_2 > 100 \]
\[ \text{infeasible} \]

\[ \text{x}_2 \text{ is significant, but this question is not so good} \]

\[ \text{LP: } x_1 = 0 \]
\[ x_2 = 7.505 \]
\[ \text{Obj: } 7505 \]

\[ x \leq 30 \]
\[ x \leq 50 \]
\[ x_1 \leq 49 \]
\[ x_1 = 50 \]

\[ x \geq 31 \]
\[ x_1 \geq 51 \]

\[ \text{Say we suspect } x_1 = 50, \ x_2 = 7 \]
\[ \text{is best} \]

\[ x_1 \text{ is much less significant} \]