(1) Consider the linear program max $\vec{c}^T \vec{x}$ subject to $A\vec{x} \leq \vec{b}$ and $\vec{x} \geq 0$, where

$$\vec{c} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 25 \\ 7 \\ 8 \end{bmatrix}.$$ 

You run the simplex method on this LP and obtain the final dictionary:

- $x_1 = 1 + x_4 - x_5$
- $x_2 = 6 - 2x_4 + x_5$
- $x_3 = 6 + 5x_4 - 2x_5$
- $z = 22 - 2x_4 - x_5$

(a) Put this into an LP optimization software, and verify that your software works on this example. Print out your LP description, and the output of the software; for example, if you use Gurobi, then print out the file describing the LP and the optimal solution Gurobi finds as well as the values of the $x_1, x_2$.

**Solution:** See gurobi files at the end.

(b) Change the constraint $x_1 + x_2 \leq 7$ to $x_1 + x_2 \leq 7.01$, and run your optimization software again. What is the new optimum $z$ value and optimum solution $(x_1, x_2)$?

**Solution:** $z = 22.02$, $x_1 = 0.99$, $x_2 = 6.02$.

How could you have predicted this from the dictionary?

**Solution:** Since $x_4$ is the slack variable corresponding to the inequality $x_1 + x_2 \leq 7$, the change of 0.01 increases the objective $z$ by 0.02, corresponding to the coefficient of $-2$ in the optimal dictionary line $z = 22 - 2x_4 - x_5$; similarly the $x_4$ coefficients of

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1 and $-2$ in this dictionary for $x_1$ and $x_2$ (respectively) account for the change of $-0.01$ and $0.02$ in these variables new values in the optimal solution.

(c) Same question where the constraint changes to $x_1 + x_2 \leq 6.99$.

**Solution:** When we run our software we get $z = 21.98$, $x_1 = 1.01$, $x_2 = 5.98$. Now we have that the change in the $x_4$ constraint is $-0.01$, which is why we see the changes in $z, x_1, x_2$ of, respectively, $-0.02, 0.01, -0.02$.

(d) Same question where you leave $x_1 + x_2 \leq 7$, but now change the first constraint to $x_1 + 3x_2 \leq 25.01$.

**Solution:** Your software should show the same optimum solution as in part (a). This is because the change of 25 to 25.01 corresponds to a change in $x_3$, which is a constraint that is not active in part (a), i.e., that is satisfied with strict inequality.

(2) Use your software to solve the linear program: maximize $x_1$ subject to $x_1 \geq 4$, $x_1 \leq 3$, and $x_1 \geq 0$. Print its output and make sure that your software says that the above linear program is infeasible.

**Solution:** See the Gurobi file and output for this exercise.