DualityGap = (ValueBettyAnnouncesPure) − (ValueAliceAnnouncesPure)

If $A$ is a $2 \times 2$ matrix, then either (1) the duality gap is zero, or (2) Alice and Betty have mixed strategies where the values are balanced, e.g.,

$$[x_1 \quad 1 - x_1]A = [v \quad v]$$

for Alice.

LP standard form: maximize $\vec{c} \cdot \vec{x}$, subject to $A\vec{x} \leq \vec{b}$, $\vec{x} \geq \vec{0}$.

Unbounded LP: A variable enters, but nothing leaves.

2-phase method: (1) introduce $x_0$ on right, (2) pivot $x_0$ into the basis for a feasible dictionary, and try to maximize $w = -x_0$, (3) if $w$ reaches 0, pivot $x_0$ out of dictionary and eliminate all $x_0$; e.g.,

$x_4 = -7 + \cdots + x_0$

$x_9 = -8 + \cdots + x_0$  

$x_0$ enters, $x_9$ leaves

The formulas for simplex method dictionaries (in standard form) is

$$\vec{x}_B = A_B^{-1}\vec{b} - A_B^{-1}A_N\vec{x}_N$$

$$z = \vec{c}_B^T A_B^{-1} \vec{b} + (\vec{c}_N^T - \vec{c}_B^T A_B^{-1}A_N)\vec{x}_N$$

In the computation above, we compute $\vec{c}_B^T A_B^{-1}A_N$ by first computing $\vec{c}_B^T A_B^{-1}$, and then multiplying the result (a row vector) times $A_N$; it would be more expensive to first compute $A_B^{-1}A_N$.

For the $A_B^{-1}$ of the $i - 1$-th and $i$-th dictionaries we have

$$A_B^{-1} = E_i^{-1}A_{B_{i-1}}$$

where $E_i$ is an eta matrix, equal to the identity except in one column. This formula can be applied recursively to get

$$A_{B_{i+k}}^{-1} = E_{i+k}^{-1}E_{i+k-1}^{-1} \cdots E_i^{-1}A_{B_{i-1}}^{-1}.$$

Let the $b$-th row in a matrix game be $\vec{f}(b)$. If $\vec{f}$ is a convex function (i.e., concave up), then Alice has an optimal strategy that is some combination of the smallest and largest values of $b$ (i.e., the top and bottom rows). If $\vec{f}$ is concave down, then Alice has an optimal strategy this is some combination of two adjacent rows. (These combinations can be 100% of one row in certain cases.)