\[ x_B = A_B^{-1} \left( \sum_{i} A_N x_N \right) \]
\[ 2 = \sum_{i} C_N^T \left( a^T A_B a_N \right) x_N \]

Modify \( C \) slightly, expect same final dictionary.
--- some basic \( x_B \) non-basic \( x_N \)

If \( C \) changes slightly

\[ B = \left\{ \begin{array}{c}
A_B, A_N \\
\end{array} \right\} \]

In practice: change \( C \), just need to recompute \( z \) raw

max \( 4x_1 + 5x_2 \)

s.t. \( x_1 + 2x_2 \leq 8 \)
\( x_1 + x_2 \leq 5 \)
\( 2x_1 + x_2 \leq 8 \)
\( x_1, x_2 \geq 0 \)

\( X_1 = \left\{ \begin{array}{c}
2 \\
3 \\
1 \\
\end{array} \right\} \)
\( X_2 = \left\{ \begin{array}{c}
5 \\
\end{array} \right\} \)
\( X_3 = \left\{ \begin{array}{c}
0.01 \\
\end{array} \right\} \)
\( z = 23 \)

Math 340, Nov 27

- Sensitivity Analysis
  
  \( z \) for \( \beta \) decl

- Today & Monday:
  
  Poker game \( 2^{52} \) strategies
  
  \( (\text{Convexity}) \)
  
  Take solved LP, solved by simplex method

if \( C \) changes slightly beyond the point where \( x_B, x_N \) are still a final dictionary

\[ X_B = \]
\[ z = 36 - 2x_2 - 5x_3 + 0.01x \]

Expect only one pivot away from optimal...

\( \Rightarrow \)

If \( \beta \) changes \( \ldots \) 😞
Dual dictionary

\[ x_1 = 2 \]
\[ x_2 = 3 \]
\[ x_3 = 1 \]
\[ z = 2z - \_ \]

\[ y_1 = 1 + \text{etc.} \]
\[ y_2 = 3 + \text{etc.} \]
\[ w = -2z - y_3 - 2y_4 - 3y_5 \]

Dual LP:

\[ (y_1) \quad 4 \leq y_1 + y_2 + 2y_3 \]
\[ (y_5) \quad 5 \leq 2y_1 + y_2 + y_3 \]
\[ w = -8y_1 - 5y_2 - 8y_3 \]

Dual pivot = simplex method pivot in the dual

Simplex Duality Dual pivot

\[ x_1 = 3 \]
\[ x_2 = 2 \]
\[ x_3 = 1 \]
\[ z = -3x_1 + 5x_2 + 2x_4 \]
\[ = 3 \]
\[ = 5 \]
\[ = 2 \]
\[ = \_3y_1 - 2y_4 + y_5 \]
Similarly:
- optimal dictionary can usually add a new decision variable easily
  in simplex method
  \[ \text{max } 4x_1 + 5x_2 \]

\[
\begin{align*}
    \text{max } & \quad 4x_1 + 5x_2 + 2x_3 \\
     \iff & \quad \text{usually adding a inequality can be more difficult}
\end{align*}
\]

- adding a variable

\[ y_4 = -4 + y_1 + y_2 + 2y_3 \geq 0 \]
\[ y_5 = -5 + 2y_1 + y_2 + y_3 \geq 0 \]

\[ w = -8y_1 - 5y_2 - 8y_3 \]
\[ \text{not feasible} \]
\[ y_2 = 0, \quad y_1 = y_3 = 0 \]
\[ w = -50 \]
(claim \( w^* = -23 \))

Poker Game: 52 strategies

Alice & Betty put 1 penny each into the middle

Alice draws a card

(Old! Black or Red)

60%, 50%

New:
- high card #52, next highest 1 \#51
- A Spades, A Heart
- 2nd to lowest, lowest

\[ \begin{align*}
    (x_1 + 2x_2 & \leq 8) & y_1 \\
    (x_1 + x_2 & \leq 5) & y_2 \\
    (2x_1 + x_2 & \leq 8) & y_3 \\
    (3x_1 + 4x_2 & \leq 12) & (\text{new } y_4)
\end{align*} \]

Theme: 2-for-1 in simplex method via duality,

\[ \implies \text{Game theory} \]
Alice draws card # C

$C = 52, 51, ..., 1$

Betty wins: $\frac{51 - (C - 1)}{51}$

Alice wins: $\frac{C - 1}{51}$

= Betty: Calls or Folds

Alice: Ace Spades: Fold or Bet
      Ace Hearts: Fold or Bet

2 choices for 52 cards
  lowest, 2: Fold or Bet

Alice looks at card:
folds or bets (1 penny)

Betty (no info) (draws a card)
folds or calls

Alice sees Ace Spades
51 cards left:

Alice wins $\frac{51}{51}$ times

Alice sees Ace hearts

wins $\frac{50}{51}$ times

\[
\begin{pmatrix}
1 & 2 \\
3 & 1 \\
5 & 6 \\
7 & 10 \\
10 & 7 \\
\end{pmatrix}
\]

Large Areas

Folds

\(\ldots\)

(1, 2)

(3, 1)

Calls

Betty

Cell

Fold

Alice Strategy

\[
\begin{pmatrix}
0 & 0 \\
\end{pmatrix}
\]