MATH 516-101
INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS (I)

Term 1 (Sept-Dec 2022)

http://www.math.ubc.ca/∼jcwei

Instructor: Juncheng Wei, LSK 303B, Tel. 604-822-6510, E-mail: jcwei@math.ubc.ca

Time and Place: Mon-Wed-Fri: 1pm to 2pm in MATH-204

Content: This course is an introduction to the qualitative theory of partial differential equations (PDEs). It should be useful to students with interests in applied mathematics, differential geometry, mathematical physics, fluid mechanics, mathematical biology, probability, harmonic analysis, dynamical systems, and other areas, as well as to PDE/Analysis-focused students. We will review and expose a few analytic tools along the way, e.g. Fourier transform and weak convergence.

Prerequisites: Basic real analysis, including convergence, Lebesgue integral and L^p spaces.

Topics:

- 1. Classical linear equations
  - (a) Laplace equation I: mean value properties and maximum principles, applications to uniqueness and regularity
  - (b) Laplace equation II: regularity of weakly harmonic functions, analyticity, Harnack inequality, etc
  - (c) Laplace equation III: existence: Perrons sub-solution method and Dirichlet principle
  - (d) Heat equations; their solution formulas and Maximum Principles; Tokonov's example
  - (e) Wave equation: d'Alembert's formula and Kirchhoff's formula
- 2. Sobolev spaces
  - (a) weak derivatives and Sobolev spaces
  - (b) inequalities of Sobolev, Morrey, Poincare, and Gagliardo-Nirenberg; compactness and embeddings
  - (c) approximations, extensions, trace, compactness, and dual spaces
- 3. Weak solutions of elliptic equations
  - (a) weak solutions and maximal principle
  - (b) existence and eigenvalues by Lax-Milgram theorem and Fredholm alternative
  - (c) regularity
  - (d) application to semilinear elliptic problems

  - (e) analogous results for 2nd-order parabolic equations
- 4. Classical solutions of second order elliptic equations (4 hours)
  - (a) weak and strong maximal principles
  - (b) Holder spaces and Schauders a priori estimates
  - (c) existence by the method of continuity
- 6. Variational and non-variational techniques for nonlinear PDEs
  - (a) Direct method of calculus of variations
  - (b) Constraint methods
  - (c) Mountain-pass theorem
  - (d) Fixed point methods
  - (e) Method of sub-super solutions
  - (f) Monotonicity formula (Almgren, ALt-Caffarelli-Friedman) and Pohozaev identities

References: We will mainly follow Evanss book and also use materials from the others.

1. Main Book: Partial Differential Equations, 2nd ed., by L. C. Evans, American Math Society, 2010. See authors homepage http://math.berkeley.edu/ evans for errata. This is a general text suitable for a first course and also for reference.

2. Reference book I: Partial Differential Equations, 4th ed., by Fritz John, Springer-Verlag. This is a classic textbook and contains materials not found elsewhere, e.g. Weyls lemma and extended treatise on wave equation.

3. Reference book II: Elliptic Partial Differential Equations of Second Order, 2nd ed., by David Gilbarg and Neil S. Trudinger, Springer-Verlag, Classics in Mathematics series. This book is specialized in elliptic equations and is a standard reference.

4.Reference book III: Elliptic Partial Differential Equations, by Qing Han and Fanghua Lin, Courant lecture Notes. This book is easy to read and cover most of the materials in book 3.

5. Reference book IV: Partial Differential Equations: A First Course, Rustum Choksi, AMS. This paper has many good materials for the first and second part of the course.

In each week, I will pinpoint to the exact chapter of the book.

Office Hour: MWF, 3:30-5pm. Zoom Office Hours will be announced.
Zoom Office Hour: W, 8:30-10pn

Assessment: The grade is based on homework assignments (five or six).