MATH400-101 Homework Assignment 1 (Due Date: September 16 , 2022)

Homework is admitted until 1am on September 16, 2022. You can submit either at my office by 6pm or at Canvas. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. (10points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$(a)\ u_x = 0, u(x,x) = x^2 - 1; \quad (b)\ u_x = 0, u(x,x^3) = e^x; \quad (c)\ u_x = 0, u(x,x^2) = e^{x^2}$$

2. (10points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$3u_x + 5u_y = 0, u(x,-x) = x^2 - 1$$

3. (10points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$2yu_x + u_y = 0, u(0,y) = e^y$$

4. (20points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$u_x + 2xyu_y = u, u(x,1) = x^3, 0 \le x \le 1$$

5. (10points) Solve $u_t + 2u_x = 0$ for $x > 0, t > 0$ with $u(0,t) = e^t, t > 0$ and $u(x,0) = x^2, x > 0$.
6. (10points) Solve $xu_t + tu_x = 0$ for $x > 0, t > 0$ with $u(0,t) = t^2, t > 0$ and $u(x,0) = x, x > 0$.
7. (15points) Find the solution to the following first order PDE

$$2u_x - 3u_y + u = e^x, u(x,x) = x, -\infty < x < +\infty$$

8. (15points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$xu_x + (x+y)u_y = u + x, u(x,3x) = 0, -\infty < x < +\infty$$

Useful technique for first order ODE:

$$\frac{dy}{dt} + ay = g(t)$$

Method I (Multiplier):

$$e^{at}\left(\frac{dy}{dt} + ay\right) = \frac{d}{dt}(e^{at}y) = e^{at}g(t)$$

$$y = Ce^{-at} + e^{-at}\int e^{a\tau}g(\tau)d\tau$$

Method II (Undetermined Coefficients): $y = y_p + Ce^{-at}$ where y_p is a special solution.