1. (10 points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$u_x + 3u_y = 0, u(x, 2x) = x^2 - 1$$

2. (10 points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$yu_x + 3x^2u_y = 0, u(0, y) = y^3$$

3. (10 points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$u_x + 2yu_y = u, u(x, 1) = e^{2x}$$

4. (10 points) Solve the following first order PDE and find where the solution is defined in the $x-y$ plane.

$$xu_x + (x + y)u_y = u + x, u(x, 3x) = 0, -\infty < x < +\infty$$

5. (20 points) Solve $xu_t + tu_x = 0$ for $x > 0, t > 0$ with $u(0, t) = t^2, t > 0$ and $u(x, 0) = x, x > 0$.

6. (20 points) Solve the following first order PDE and find where the solution becomes unbounded in the $x-y$ plane.

$$u_x + e^x u_y = u^3, u = 1 \text{ on the curve } y = 2e^x, 0 \leq x \leq 1$$

7. (20 points) Let $u(x, y)$ solve the first order PDE

$$yu_x + 2x^3u_y = y^3u$$

(a) Find the general solutions. (b) Suppose we put $u = h(x)$ on $y = x^2$. Derive the condition that $h(x)$ must satisfy for a solution to exist.