MATH305-201-2021/2022 Homework Assignment 4 (Due Date: Feb. 7, 2022)

10pts each

1. Find a conformal mapping from the following set onto the upper half plane $S' = \{(u, v) \mid v > 0\}$:
   (a) $S = \{x > 0, -\frac{\pi}{2} < y < \frac{\pi}{2}\}$; (b) $S = \{-1 < x < 3, y > 0\}$
   Hint: use the linear map $az + b$ and $\sin(z)$.
2. Evaluate the following
   (a) $log(i)$; (b) $Log(\sqrt{3} - i)$; (c) $log(e^{1+i})$; (d) $e^{log(1+i)}$
3. Find all values of
   (a) $e^z = -1 - i$; (b) Principal Values of $(1 + i)^i$; (d) $i^{\frac{1}{3}}$
4. Solve the following equations
   (a) $Log(z^2 - 1) = \frac{i\pi}{2}$; (b) $e^{2z} + e^z + 1 = 0$; (c) $z^{\frac{1}{2}} + 1 - i = 0$ (here $z^{\frac{1}{2}}$ denotes the principal branch)
5. Determine the domain of analyticity (branch cut) of
   (a) $Log(1 + z^2)$; (b) $Log(\frac{1-z}{1+z})$
6. Which of the followings are true statements? For the ones that are false find a counterexample
   (a) $e^{log(z)} = z$; (b) $e^{Log(z)} = z$; (c) $Log(e^z) = z$; (d) $log(e^z) = z$; (e) $log(z_1 z_2) = logz_1 + logz_2$; (f) $log(z) = -log(\frac{1}{z})$; (g) $log(z^{\frac{1}{2}}) = \frac{1}{2}log(z)$
7. Find a branch cut of $log(z - 1)$ that is analytic at all points in the plane except those on the following rays.
   (a) $\{x \leq 1, y = 0\}$; (b) $\{x \geq 1, y = 0\}$; (c) $\{x = 1, y \geq 0\}$
8. Find a branch cut for $\sqrt{z(z-1)}$ that is analytic in $C\backslash[0, 1]$ and takes value $\sqrt{2}$ at $z = 2$.
9. Determine a branch of $log(z^2 + 2z + 2)$ that is analytic at $z = -1$ and takes value 0 at $z = -1$, and find its derivative there.
10. Determine a branch of $log(1 + z^2)$ that is analytic at $z = 0$ and takes the value $2\pi i$ there.