SOLUTION TO QUIZ 1

Question 1. Find a parametrization of the intersection of $x^2 + y^2 = 4$ and the surface $z = xy$.

Solution: Any points on the cylinder $x^2 + y^2 = 4$ can be written as $(2 \cos t, 2 \sin t, z)$. If the point is also on the surface $z = xy$, then we must have $z = (2 \cos t)(2 \sin t) = 4 \sin t \cos t$. Therefore, the parametrization is given by $(2 \cos t, 2 \sin t, 4 \sin t \cos t)$.

Question 2. At what point do the curves $r_1(t) = (t, 1-t, 3+t^2)$ and $r_2(s) = (3-s, s-2, s^2)$ intersect? Find their angle of intersection.

Solution: At the intersection, we have $r_1(t) = r_2(s)$, so that we have 3 equations:

\[
\begin{align*}
    t &= 3 - s \\
    1-t &= s - 2 \\
    3 + t^2 &= s^2
\end{align*}
\]

Solving the equations, we get $t = 1$ and $s = 2$. Therefore, the intersection is $r_1(1) = r_2(2) = (1, 0, 4)$.

The angle of intersection is the angle between the tangent vectors. Since $r_1'(t) = (1, -1, 2t)$ and $r_2'(s) = (-1, 1, 2s)$, the tangent vector to the curve $r_1(t)$ at $(1, 0, 4)$ is $r_1'(1) = (1, -1, 2)$ and the tangent vector to the curve $r_2(s)$ at $(1, 0, 4)$ is $r_2'(2) = (-1, 1, 4)$. If the angle between them is $\theta$, then

\[
\cos \theta = \frac{r_1' \cdot r_2'}{|r_1'| |r_2'|} = \frac{(1, -1, 2) \cdot (-1, 1, 4)}{|(1, -1, 2)| |(-1, 1, 4)|} = \frac{1}{\sqrt{3}}
\]

Therefore, $\theta = \cos^{-1} \frac{1}{\sqrt{3}}$.

Question 3. Suppose you start at the point $(0, 0, 3)$ and move 5 units along the curve $r(t) = (3 \sin t, 4t, 3 \cos t)$ in the positive direction, where are you now?

Solution. Note that $r(0) = (0, 0, 3)$. As we are moving 5 units in the positive direction along the curve, the destination would be $r(T)$ with $T$
satisfying $5 = \int_0^T |\mathbf{r}'(t)| \, dt$. Since $\mathbf{r}'(t) = (3 \cos t, 4, -3 \sin t)$, $|\mathbf{r}'(t)| = 5$. Therefore, $5 = \int_0^T 5 \, dt = 5T$ and hence $T = 1$. The destination is $\mathbf{r}(1) = (3 \sin 1, 4, 3 \cos 1)$. 