[15] 1) Let \( C \) be the curve given by
\[
\vec{r}(t) = t \cos t \hat{i} + t \sin t \hat{j} + t^2 \hat{k}, \quad 0 \leq t \leq \pi
\]
a) Find the unit tangent to \( C \) at the point \((-\pi, 0, \pi^2)\).
b) Calculate the line integral
\[
\int_C \sqrt{x^2 + y^2} \, ds
\]
c) Find the equation of a smooth surface in 3-space containing the curve \( C \).
d) Sketch the curve \( C \).

[15] 2) Let \( C \) be the part of the curve of intersection of \( xyz = 8 \) and \( x = 2y \) which lies between the points \((2, 1, 4)\) and \((4, 2, 1)\). Calculate
\[
\int_C \vec{F} \cdot d\vec{r}
\]
where
\[
\vec{F} = x^2 \hat{i} + (x - 2y) \hat{j} + x^2 y \hat{k}
\]

[15] 3) Let \( S \) be spherical cap which consists of the part of the sphere \( x^2 + y^2 + (z - 2)^2 = 4 \) which lies under the plane \( z = 1 \). Let \( f(x, y, z) = (2 - z)(x^2 + y^2) \). Calculate
\[
\iint_S f(x, y, z) \, dS
\]

[15] 4) Let \( S \) be the part of the surface \((x + y + 1)^2 + z^2 = 4 \) which lies in the first octant. Find the flux of \( \vec{F} \) downwards through \( S \) where
\[
\vec{F} = xy \hat{i} + (z - xy) \hat{j}
\]

[20] 5) State the divergence theorem.
Let \( D \) be the cylinder \( x^2 + y^2 \leq 1, \quad 0 \leq z \leq 5 \). Calculate the flux of the vector field
\[
\vec{F} = (x + y e^z) \hat{i} + \frac{1}{2} y^2 z e^z \hat{j} + (3z - y z e^z) \hat{k}
\]
outward through the curved part of the surface of \( D \).

[20] 6) Let \( S \) be the part of the surface \( z = 16 - (x^2 + y^2)^2 \) which lies above the \( xy \)-plane. Let \( \vec{F} \) be the vector field
\[
\vec{F} = x \ln(1 + z) \hat{i} + x(3 + y) \hat{j} + y \cos z \hat{k}
\]
Calculate
\[
\iint_S \nabla \times \vec{F} \cdot \hat{n} \, dS
\]
where \( \hat{n} \) is the upward normal on \( S \).

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