MATHEMATICS 317 April 2000 Final Exam

[21] 1) A particle moves along the curve $C$ of intersection of the surfaces $z^2 = 12y$ and $18x = yz$ in the upward direction. When the particle is at $(1, 3, 6)$ its velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ are given by

$$\mathbf{v} = 6\mathbf{i} + 12\mathbf{j} + 12\mathbf{k} \quad \mathbf{a} = 27\mathbf{i} + 30\mathbf{j} + 6\mathbf{k}$$

a) Write a vector parametric equation for $C$ using $u = \frac{y}{6}$ as a parameter.
b) Find the length of $C$ from $(0, 0, 0)$ to $(1, 3, 6)$.
c) If $u = u(t)$ is the parameter value for the particle’s position at time $t$, find $\frac{du}{dt}$ when the particle is at $(1, 3, 6)$.
d) Find $\frac{d^2u}{dt^2}$ when the particle is at $(1, 3, 6)$.

[20] 2) The vector field $\mathbf{F}(x, y, z) = Ax^3y^2z\mathbf{i} + (z^3 + Bx^4yz)\mathbf{j} + (3yz^2 - x^4y^2)\mathbf{k}$ is conservative on $\mathbb{R}^3$.
a) Find the values of the constants $A$ and $B$.
b) Find a scalar field $\phi$ such that $\mathbf{F} = \nabla \phi$ on $\mathbb{R}^3$.
c) If $C$ is the curve $y = -x$, $z = x^2$ from $(0, 0, 0)$ to $(1, -1, 1)$, evaluate $I = \int_C \mathbf{F} \cdot d\mathbf{r}$.
d) Evaluate $J = \int_C (z - 4x^3y^2z)dx + (z^3 - x^4yz)dy + (3yz^2 - x^4y^2)dz$, where $C$ is the curve of part (c).
e) Let $T$ be the closed triangular path with vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, oriented counterclockwise as seen from the point $(1, 1, 1)$. Evaluate $\int_T (\mathbf{F} + \mathbf{F}) \cdot d\mathbf{r}$.

[17] 3) Let $R$ be the region in the first quadrant of the $xy$–plane bounded by the coordinate axes and the curve $y = 1 - x^2$. Let $C$ be the boundary of $R$, oriented counterclockwise.
a) Evaluate $\int_C x \, ds$.
b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = (\sin(x^2) - xy)\mathbf{i} + (x^2 + \cos(y^2))\mathbf{j}$.

[17] 4) Let $S$ be the part of the surface $x^2 + y^2 + 2z = 2$ that lies above the square $-1 \leq x \leq 1$, $-1 \leq y \leq 1$.
a) Find $\iint_S \frac{x^2 + y^2}{x^2 + y^2 + 2z} \, dS$.
b) Find the flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ upward through $S$.

[15] 5) A smooth surface $S$ lies above the plane $z = 0$ and has as its boundary the circle $x^2 + y^2 = 4y$ in the plane $z = 0$. This circle also bounds a disk $D$ in that plane. The volume of the 3–dimensional region $R$ bounded by $S$ and $D$ is 10 cubic units. Find the flux of the vector $\mathbf{F}(x, y, z) = (x + x^2y)\mathbf{i} + (y - xy^2)\mathbf{j} + (z + 2x + 3y)\mathbf{k}$ through $S$ in the direction outward from $R$.

[10] 6) An object moves along a curve in the $xy$–plane having polar equation $r = \frac{1}{\sqrt{\rho}}$ (where $\alpha$ is a constant) under the influence of a central force so that the object has no transverse acceleration.
a) Verify that $r^2\dot{\theta} = h$ remains constant as the object moves.
b) Express the magnitude of the acceleration of the object as a function of $r$ and $h$.

We did not cover the material in question 6 this year.