Quiz 5 Math 200

Problem (1) Find the surface area of the part of the surface $z = y^2 - x^2$ which lies between the cylinders $x^2+y^2=1$ and $x^2+y^2=4$.

Problem (2) Compute $\iiint_E x \, dVol$ where E is the solid regim bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.

Problem 1 solution $f(x,y) = y^2 - x^2$ $f_x = -2x$ $f_y = 2y$

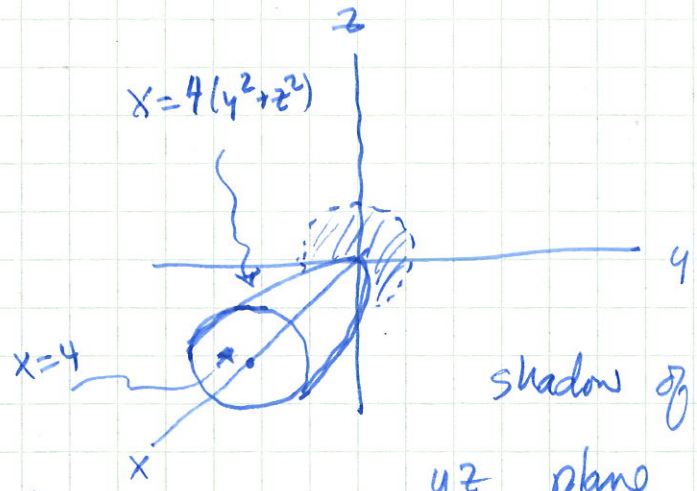
$$\text{Area} = \iint_{1 \le x^2+y^2 \le 4} \sqrt{1+f_x^2+f_y^2}\, dx\,dy = \iint_{1 \le x^2+y^2 \le 4} \sqrt{1+4(x^2+y^2)}\, dx\,dy$$

$$= \int_{\theta=0}^{2\pi} \int_{r=1}^{2} \sqrt{1+4r^2}\, r\, dr\, d\theta = 2\pi \left[\frac{1}{8} \cdot \frac{2}{3} (1+4r^2)^{3/2} \right]_1^2$$

$$= \frac{\pi}{6}\left[17^{3/2} - 5^{3/2} \right] = \frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5} \right)$$

Problem 2 solution

$$\iiint_E x \, dVol =$$



shadow of E on yz plane is $y^2+z^2 \le 1$

$$= \iint_{y^2+z^2\le 1} \left(\int_{x=4(y^2+z^2)}^{4} x \, dx \right) dy\,dz$$

$$= \iint_{y^2+z^2\le 1} \left[\frac{x^2}{2} \right]_{4(y^2+z^2)}^{4} dy\,dz = \iint_{y^2+z^2\le 1} \left(8 - 8(y^2+z^2)^2 \right) dy\,dz$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{1} \left(8 - 8(r^2)^2 \right) r \, dr\, d\theta$$

$$= 2\pi \left[8 \int_0^1 (r - r^5)\, dr \right] = 16\pi \left[\frac{r^2}{2} - \frac{r^6}{6} \right]_0^1$$

$$= 16\pi \left(\frac{1}{2} - \frac{1}{6} \right) = \frac{16\pi}{3}$$