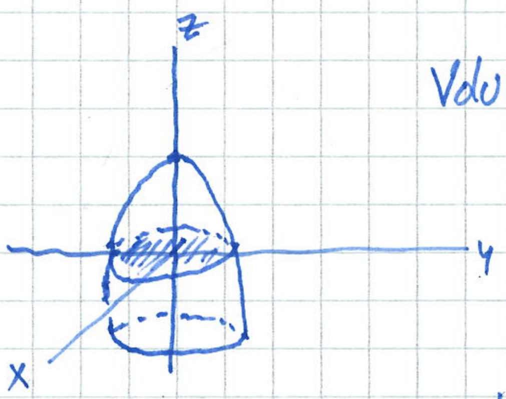
Quiz 4 math 200-106 SOLUTIONS

problem 1. Find the volume of the solid region lying below the paraboloid $z = 2 - 4x^2 - 4y^2$ and above the xy plane



$$\text{Volume} = \iint_R (2 - 4x^2 - 4y^2)\,dxdy$$

where R is the region in the xy plane contained inside of the curve formed by the intersection of the paraboloid and the xy plane.

$z = 2 - 4x^2 - 4y^2$ & $z = 0 \Rightarrow 4x^2 + 4y^2 = 2$

$\Rightarrow x^2 + y^2 = \frac{1}{2}$ $\quad R = \left\{ (x,y) : x^2 + y^2 \leq \frac{1}{2} \right\}$

In polar coordinates, $R = \left\{ (r,\theta) : r \leq \frac{1}{\sqrt{2}} \right\}$

$2 - 4x^2 - 4y^2 = 2 - 4r^2$, and $dxdy = r\,dr\,d\theta$ so

$$\text{Volume} = \int_{\theta=0}^{2\pi} \int_{r=0}^{1/\sqrt{2}} (2 - 4r^2)\, r\,dr\,d\theta$$

$$= 2\pi \int_{r=0}^{1/\sqrt{2}} (2r - 4r^3)\,dr = 2\pi \left[r^2 - r^4 \right]_0^{1/\sqrt{2}}$$

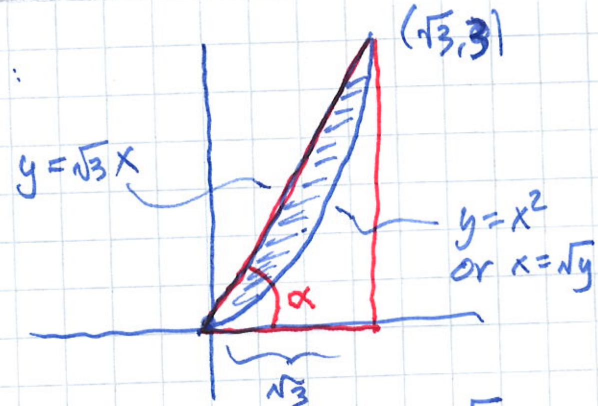
$$= 2\pi \left(\frac{1}{2} - \frac{1}{4} \right) = \frac{\pi}{2}$$

Problem 2 Let R be the region in the plane between the line $y = \sqrt{3}x$ and the parabola $y = x^2$.

Rewrite the double integral $\iint_R f(x,y)\,dA$

as
1. An iterated integral where x is integrated first.
2. " " " " y " " ".
3. " " " in polar coordinates.

Solution:



curves intersect where

$\sqrt{3}x = x^2 \Rightarrow x = 0$ or $\sqrt{3}$

$(0,0)$ $(\sqrt{3},3)$

$$\cos\alpha = \frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2+3^2}} = \frac{\sqrt{3}}{\sqrt{12}} = \frac{1}{2} \quad \text{so} \quad \alpha = \frac{\pi}{3}$$

$$\iint_R f\,dA = \int_{y=0}^{3}\int_{x=\frac{y}{\sqrt{3}}}^{\sqrt{y}} f(x,y)\,dx\,dy$$

$$= \int_{x=0}^{\sqrt{3}}\int_{y=x^2}^{\sqrt{3}x} f(x,y)\,dy\,dx$$

$$= \int_{\theta=0}^{\pi/3}\int_{r=0}^{\tan\theta\sec\theta} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

$y = x^2$ in polar:

$r\sin\theta = r^2\cos^2\theta$

$r = \frac{\sin\theta}{\cos^2\theta} = \tan\theta\sec\theta$