problem 1. Find the volume of the solid region lying below the paraboloid \( z = 2 - 4x^2 - 4y^2 \) and above the xy plane.

\[
\text{Volume} = \iiint_{R} (2 - 4x^2 - 4y^2) \, dx \, dy \\
\text{where } R \text{ is the region in the xy plane contained inside of the curve formed by the intersection of the paraboloid and the xy plane.}
\]

\[z = 2 - 4x^2 - 4y^2 \quad \Rightarrow \quad 4x^2 + 4y^2 = 2\]
\[\Rightarrow \quad x^2 + y^2 = \frac{1}{2}\]

\[R = \{(x, y) : x^2 + y^2 \leq \frac{1}{2}\}\]

In polar coordinates, \( R = \{(r, \theta) : r \leq \frac{1}{\sqrt{2}}\}\). \( 2 - 4x^2 - 4y^2 = 2 - 4r^2 \), and \(dx \, dy = r \, dr \, d\theta\), so

\[
\text{Volume} = \int_{0}^{2\pi} \int_{0}^{\frac{1}{\sqrt{2}}} (2 - 4r^2) \, r \, dr \, d\theta
\]

\[= 2\pi \left[ \frac{r}{2} - \frac{4}{3} \right]_{r=0}^{r=\frac{1}{\sqrt{2}}}
\]

\[= 2\pi \left( \frac{1}{2} - \frac{1}{4} \right) = \frac{\pi}{2}\]
Problem 2. Let $R$ be the region in the plane between the line $y = \sqrt{3}x$ and the parabola $y = x^2$.

Rewrite the double integral $\iiint_{R} f(x,y) \, dx \, dy$ as

1. An iterated integral where $x$ is integrated first.
2. " " " " " " " " " " $y$ " " " " " in polar coordinates.

Solution:

$y = \sqrt{3}x$  \hspace{1cm} $y = x^2$

$\cos \alpha = \frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2 + 3^2}} = \frac{\sqrt{3}}{\sqrt{12}} = \frac{1}{2}$  \hspace{1cm} \text{so}  \hspace{1cm} \alpha = \frac{\pi}{3}$

$\iiint_{R} f(x,y) \, dx \, dy = \int_{0}^{3} \int_{0}^{\sqrt{3}x} f(x,y) \, dy \, dx$

$= \int_{0}^{\sqrt{3}} \int_{0}^{3} f(x,y) \, dy \, dx$

$= \int_{0}^{\pi/3} \int_{0}^{\tan \sec \theta} f(rcos\theta, rsin\theta) \, r \, dr \, d\theta$