Midterm — October 13th, 2017 Duration: 50 minutes

*This test has **4 questions** on **7 pages**, for a total of 30 points.*

- Read all the questions carefully before starting to work.
- All questions are long-answer; you should give complete arguments and explanations for all your calculations. Write legibly and in a coherent order.
- Continue on the back of the previous page if you run out of space or use the blank page at the end. If you continue a problem on a different page, indicate this clearly at the bottom of the problem's original page.
- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: Solutions Last Name: _____

Student-No: _____

Signature: _____

Question:	1	2	3	4	Total
Points:	8	5	9	8	30
Score:					

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   (i) speaking or communicating with other examination candidates, unless otherwise authorized;
   (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
   (iii) purposely viewing the written papers of other examination candidates;
   (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1. Let

$$w = f(u,v), \quad u = g(x,y), \quad v = h(x,y).$$

The following table gives the values of the various functions and their derivatives at the given values of their arguments:

	f	f_u	f_v	g	g_x	g_y	h	h_x	h_y
$(0,1)$	6	3	7	2	-3	10	3	2	-5
$(2,3)$	13	1	-1	4	-2	8	-1	3	7

3 marks (a) Compute $\frac{\partial w}{\partial x}$ at $x = 0$, $y = 1$.

$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial w}{\partial v}\frac{\partial v}{\partial x} = f_u g_x + f_v h_x$

at $(x,y) = (0,1)$, $u = g(0,1) = 2$, $v = h(0,1) = 3$, so $(u,v) = (2,3)$

$\left.\frac{\partial w}{\partial x}\right|_{(x,y)=(0,1)} = f_u(2,3)g_x(0,1) + f_v(2,3)h_x(0,1)$

$= 1\cdot(-3) + (-1)\cdot 2 = \boxed{-5}$

3 marks (b) Compute $\frac{\partial w}{\partial y}$ at $x = 0$, $y = 1$.

$\left.\frac{\partial w}{\partial y}\right|_{(x,y)=(0,1)} = f_u(2,3)g_y(0,1) + f_v(2,3)h_y(0,1)$

$= 1\cdot 10 + (-1)\cdot(-5) = \boxed{15}$

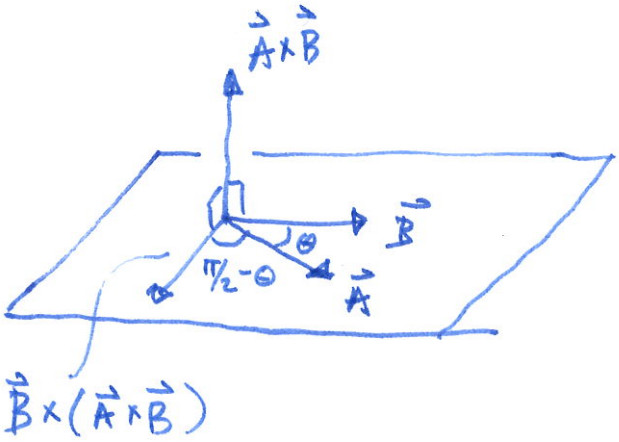
2 marks (c) Compute the directional derivative of w (as a function of x and y) at the point $(x,y) = (0,1)$ in the direction of $\vec{\mathbf{u}} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$.

$(D_u w)(0,1) = (\vec{\nabla} w)(0,1) \cdot \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

$= \langle -5, 15 \rangle \cdot \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle = -3 + 12 = \boxed{9}$

5 marks 2. Let $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ be vectors with $|\vec{\mathbf{A}}| = a$, $|\vec{\mathbf{B}}| = b$, and suppose that $\theta = \pi/3$ is the angle between them. Compute the quantity:

$$(\vec{\mathbf{B}} \times (\vec{\mathbf{A}} \times \vec{\mathbf{B}})) \cdot \vec{\mathbf{A}}$$



Using the picture:

$$(\vec{B}\times(\vec{A}\times\vec{B}))\cdot\vec{A} = |\vec{B}\times(\vec{A}\times\vec{B})||\vec{A}|\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$$

$$= |\vec{B}\times(\vec{A}\times\vec{B})|\, a\,\frac{\sqrt{3}}{2}$$

$$= |\vec{B}||\vec{A}\times\vec{B}|\cdot a\cdot\frac{\sqrt{3}}{2}$$

$$= b\cdot|\vec{A}||\vec{B}|\cdot\sin\frac{\pi}{3}\cdot a\cdot\frac{\sqrt{3}}{2}$$

$$= a^2b^2\left(\frac{\sqrt{3}}{2}\right)^2 = \boxed{\frac{3}{4}a^2b^2}$$

or Using the properties of the triple product:

$$(\vec{B}\times\vec{C})\cdot\vec{A} = \vec{B}\cdot(\vec{C}\times\vec{A}) = (\vec{A}\times\vec{B})\cdot\vec{C}$$

letting $\vec{C} = \vec{A}\times\vec{B}$ we see that

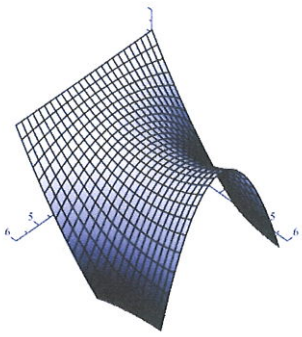
$$(\vec{B}\times(\vec{A}\times\vec{B}))\cdot\vec{A} = (\vec{A}\times\vec{B})\cdot(\vec{A}\times\vec{B}) = |\vec{A}\times\vec{B}|^2$$

$$= (ab\sin\theta)^2 = a^2b^2\left(\frac{\sqrt{3}}{2}\right)^2$$

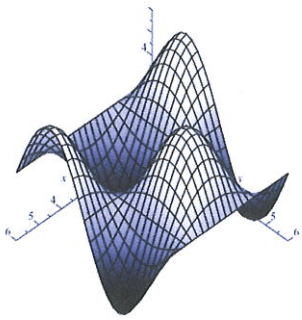
$$\boxed{= \frac{3}{4}a^2b^2}$$

9 marks

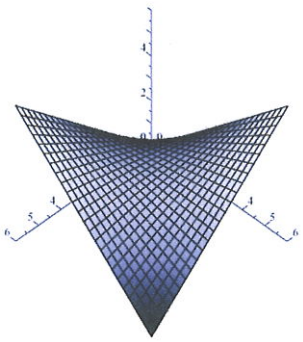
3. The axes of the nine graphs below are all oriented in the standard way: the positive x-axis is on the left, the positive y-axis is on the right, and the positive z-axis is upward. Put the letter of the corresponding contour plot from the next page in the box below each graph.



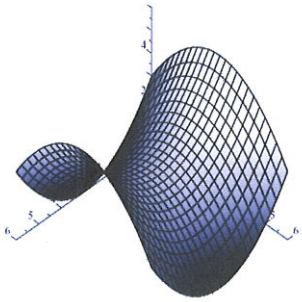
B



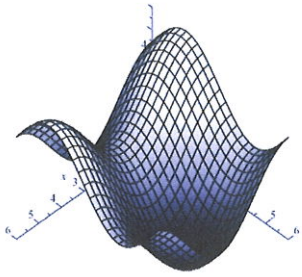
G



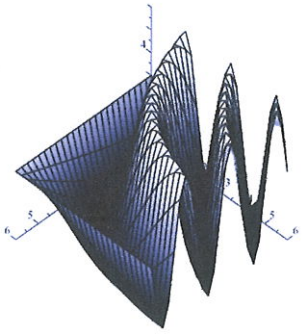
E



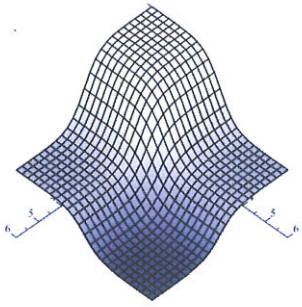
I



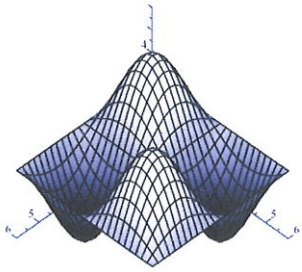
C



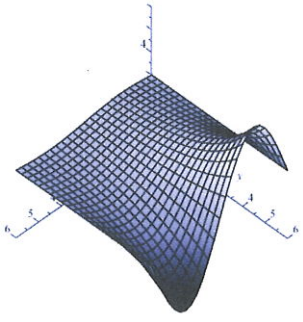
H



D

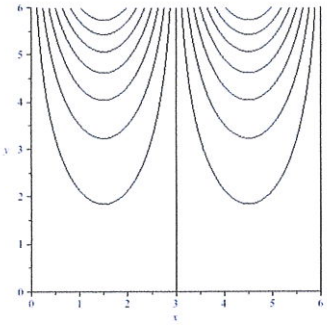


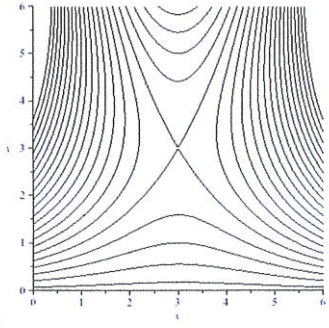
F

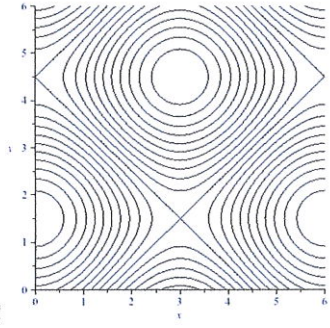


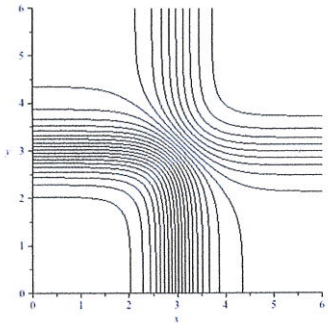
A

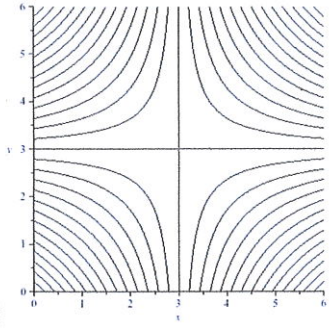
In the contour plots below, the *values* of the contours are evenly spaced. Nine of these twelve plots correspond to graphs on the previous page.

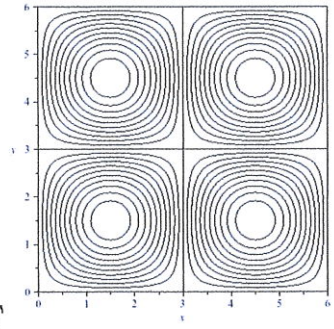
A 

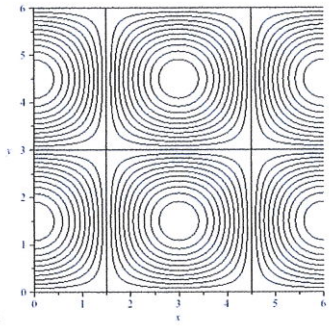
B 

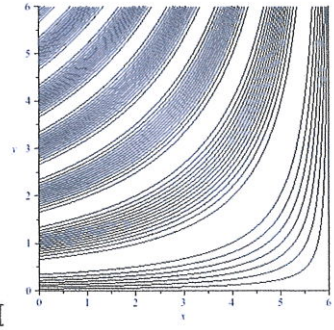
C 

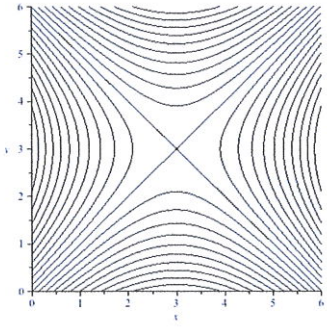
D 

E 

F 

G 

H 

I 

4 marks 4. (a) Find the equation of the plane tangent to the surface

$$z = \sqrt[3]{3x^2 + 2y^2} = f(x,y)$$

at the point $(5,5,5)$.

$z = z_0 + f_x(5,5)(x-5) + f_y(5,5)(y-5)$

$f_x = \frac{1}{3}\cdot 6x\,(3x^2+2y^2)^{-2/3}$ $\quad f_x(5,5) = 2\cdot 5\cdot\frac{1}{25} = \frac{2}{5}$

$f_y = \frac{1}{3}\cdot 4y\,(3x^2+2y^2)^{-2/3}$ $\quad f_y(5,5) = \frac{4\cdot 5}{3}\cdot\frac{1}{25} = \frac{4}{15}$

$$\boxed{z = 5 + \frac{2}{5}(x-5) + \frac{4}{15}(y-5)} \quad \text{or} \quad 15z = 75 + 6x - 30 + 4y - 20$$

$$\boxed{25 = 6x + 4y - 15z}$$

4 marks (b) Use your answer in part (a) and linear approximation to estimate the value of $\sqrt[3]{15}$ (hint: note that $3\cdot 4^2 + 2\cdot 6^2 = 120 = 2^3\cdot 15$). Write your estimate as a decimal; how close is it to the actual value of $\sqrt[3]{15} = 2.466\cdots$? Give a reason explaining the accuracy or inaccuracy of your estimate.

$f(x,y) \approx 5 + \frac{2}{5}(x-5) + \frac{4}{15}(y-5)$

since $f(4,6) = \sqrt[3]{3\cdot 4^2 + 2\cdot 6^2} = \sqrt[3]{2^3\cdot 15} = 2\sqrt[3]{15}$

we have $2\cdot\sqrt[3]{15} \approx 5 + \frac{2}{5}(4-5) + \frac{4}{15}(6-5)$

$= 5 + \frac{-2}{5} + \frac{4}{15}$

$= \frac{75-6+4}{15} = \frac{73}{15}$

so $\sqrt[3]{15} \approx \frac{73}{30} = \frac{7.3}{3}$

$$\boxed{= 2.4333\cdots}$$

$$3\overline{)7.3}\quad 2.4333\ldots$$
$$\underline{6}$$
$$13$$
$$\underline{12}$$
$$10$$

it is within 0.033... of actual value. Very accurate because (4,6) is close to (5,5)