

**Midterm 1 — May 31st, 2017      Duration: 50 minutes***This test has 4 questions on 8 pages, for a total of 25 points.*

- Read all the questions carefully before starting to work.
- All questions are long-answer; you should give complete arguments and explanations for all your calculations. Write legibly and in a coherent order.
- Continue on the back of the previous page if you run out of space or use the blank page at the end. If you continue a problem on a different page, indicate this clearly at the bottom of the problem's original page.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: Solutions Last Name: \_\_\_\_\_

Student-No: \_\_\_\_\_ Class (253 or 200): \_\_\_\_\_

Signature: \_\_\_\_\_

Question:	1	2	3	4	Total
Points:	9	4	6	6	25
Score:					

**Student Conduct during Examinations**

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
- Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  - speaking or communicating with other examination candidates, unless otherwise authorized;
  - purposely exposing written papers to the view of other examination candidates or imaging devices;
  - purposely viewing the written papers of other examination candidates;
  - using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  - using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

3 marks

1. (a) Use implicit differentiation to find  $\frac{\partial z}{\partial x}$  where  $z$  is implicitly given as a function of  $x$  and  $y$  by the equation  $xyz = e^{x+y+z}$ .

$$\frac{\partial}{\partial x}(xyz) = \frac{\partial}{\partial x}(e^{x+y+z})$$

$$yz + xy \frac{\partial z}{\partial x} = e^{x+y+z} \frac{\partial}{\partial x}(x+y+z) = (1 + \frac{\partial z}{\partial x}) e^{x+y+z}$$

$$\Rightarrow \frac{\partial z}{\partial x}(xy - e^{x+y+z}) = e^{x+y+z} - yz$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{e^{x+y+z} - yz}{xy - e^{x+y+z}}}$$

3 marks

- (b) Let  $z = e^{xy} + y$  and suppose that  $x = f(t)$  and  $y = g(t)$  where  $f$  and  $g$  are functions such that  $f(0) = 2$ ,  $f'(0) = -1$ ,  $g(0) = 3$ , and  $g'(0) = 2$ . Compute  $\frac{dz}{dt}$  when  $t = 0$ .

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

$$= ye^{xy} \frac{dx}{dt} + (xe^{xy} + 1) \frac{dy}{dt}$$

at  $t=0$   $x = f(0) = 2$   $y = g(0) = 3$   $\frac{dx}{dt} = f'(0) = -1$   $\frac{dy}{dt} = g'(0) = 2$

so  $\frac{dz}{dt} \Big|_{t=0} = 3e^6(-1) + (2e^6 + 1)2$

$$= \boxed{2 + e^6}$$



2. Label each equation by the corresponding plot.

marks

(a)  $z^2 = x^2 + y^2$ . **A**

marks

(b)  $z = x^2 - y^2$ . **E**

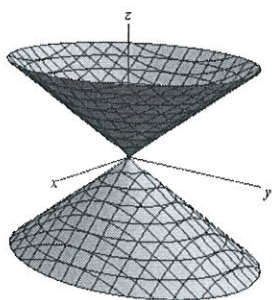
marks

(c)  $z^2 = x^2 + y^2 - 1$ . **F**

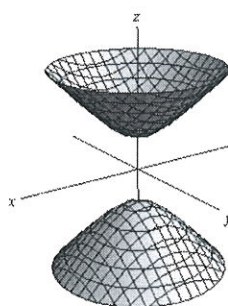
marks

(d)  $z^2 = x^2 + y^2 + 1$ . **B**

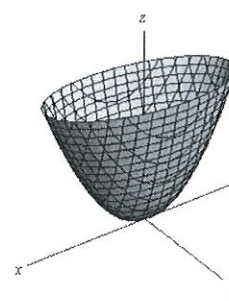
think about the trace curves  
given by  $x=0$  and  $y=0$  and  
the contour curves given by  
 $z=0$  (and  $z=C$  for  $C \neq 0$ )



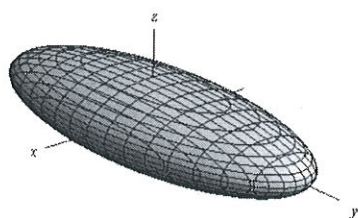
A



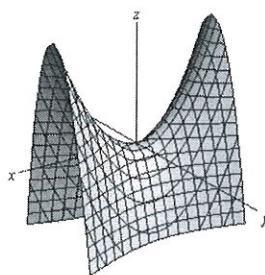
B



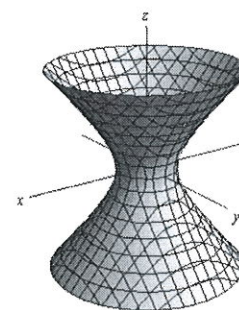
C



D



E



F



3. Let  $f(x, y) = \sqrt{xy}$ .

3 marks

(a) Find the linear approximation of  $f(x, y)$  near  $(x_0, y_0) = (40, 10)$ .

$$f(x, y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$f_x = \frac{1}{2} \frac{y}{\sqrt{xy}} \quad f_y = \frac{1}{2} \frac{x}{\sqrt{xy}} \quad f(40, 10) = 20, \quad f_x(40, 10) = \frac{1}{2} \frac{10}{20} = \frac{1}{4}$$

$$f_y(40, 10) = \frac{1}{2} \frac{40}{20} = 1$$

$$f(x, y) \approx 20 + \frac{1}{4}(x - 40) + (y - 10)$$

2 marks

2

(b) Use your answer in part (a) to find a good estimate of  $\sqrt{39}$  as a rational number (i.e. a fraction). Hint: 9 is close to 10 and is a perfect square.

$$f(39, 9) = \sqrt{39 \cdot 9} = 3\sqrt{39}$$

$$f(39, 9) \approx 20 + \frac{1}{4}(39 - 40) + (9 - 10) = 20 - \frac{1}{4} - 1 = 18\frac{3}{4}$$

$$\text{so } 3\sqrt{39} \approx 18\frac{3}{4} \Rightarrow \boxed{\sqrt{39} \approx 6\frac{1}{4}}$$

1 mark

(c) Write your estimate as a decimal number and compare it to the actual value  $\sqrt{39} = 6.2449\dots$ . How close is your estimate?

$$6\frac{1}{4} = 6.25$$

$$6.25 - 6.2449\dots = 0.0051\dots$$

we are off by less than 1%.

4. Consider the triangle given by the three points  $p_1 = (2, 1, 2)$ ,  $p_2 = (-2, 2, 1)$ , and  $p_3 = (-1, -2, 2)$ .

2 marks

- (a) Find the length of the edge from  $p_1$  to  $p_2$  and the edge from  $p_1$  to  $p_3$ .

$$\text{vector from } p_1 \text{ to } p_2 \text{ is } \langle -2, 2, 1 \rangle - \langle 2, 1, 2 \rangle = \langle -4, 1, -1 \rangle$$

$$\text{" " " " } p_3 \text{ " } \langle -1, -2, 2 \rangle - \langle 2, 1, 2 \rangle = \langle -3, -3, 0 \rangle$$

$$\text{length of edge } \overline{p_1 p_2} = |\langle -4, 1, -1 \rangle| = \sqrt{16+1+1} = \sqrt{18} = 3\sqrt{2}$$

$$\text{" " " } \overline{p_1 p_3} = |\langle -3, -3, 0 \rangle| = \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

$$\boxed{\text{both edges have length } 3\sqrt{2}}$$

1 mark

- (b) Find the angle between those two edges.

$$\langle -4, 1, -1 \rangle \cdot \langle -3, -3, 0 \rangle = (3\sqrt{2})(3\sqrt{2}) \cos \theta$$

$$12 - 3 = 9$$

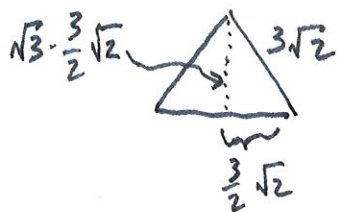
$$\text{so } 9 = 9 \cdot 2 \cos \theta \Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \boxed{\theta = \frac{\pi}{3}}$$

1 mark

(c) Find the area of the triangle.

method 1: since side lengths of 2 edges are equal with an angle of  $\pi/3$ , the triangle is equilateral:



$$\text{Area} = \frac{1}{2} \text{base} \cdot \text{height} = \frac{1}{2} (3\sqrt{2}) \left( \sqrt{3} \cdot \frac{3}{2} \sqrt{2} \right) = \boxed{\frac{9}{2}\sqrt{3}}$$

method 2:  $|\langle -4, 1, -1 \rangle \times \langle -3, -3, 0 \rangle| = \text{Area of parallelogram} = 2 \text{ Area of triangle}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 1 & -1 \\ -3 & -3 & 0 \end{vmatrix} = \langle -3, 3, 15 \rangle = 3 \langle -1, 1, 5 \rangle = 3 \sqrt{1+1+25} = 3\sqrt{27} = 9\sqrt{3}$$

so area of triangle is  $\boxed{\frac{9}{2}\sqrt{3}}$

2 marks

(d) Find the equation of the plane containing the triangle.

$\langle -4, 1, -1 \rangle \times \langle -3, -3, 0 \rangle = \langle -3, 3, 15 \rangle$  is normal to the plane  
and  $(-1, -2, 2)$  is contained in the plane. so the equation of the plane is

$$-3(x+1) + 3(y+2) + 15(z-2) = 0$$

$$\Leftrightarrow -(x+1) + (y+2) + 5(z-2) = 0$$

$$\Leftrightarrow -x-1+y+2+5z-10=0$$

$$\Leftrightarrow \boxed{-x + y + 5z = 9}$$