Problem 1 (2 points each): Let \( \vec{r} \) be the vector field

\[ \vec{r} = x \vec{i} + y \vec{j} + z \vec{k}, \]

let

\[ r = |\vec{r}|, \]

and let \( n \) be an integer. Compute the following and express your answer in terms of \( n, \vec{r}, \) and \( r \). CREDIT WILL BE GIVEN FOR THE ANSWERS ONLY, YOU MUST EXPRESS YOUR ANSWER IN TERMS OF \( n, \vec{r}, \) AND \( r \) TO RECEIVE CREDIT. THERE SHOULD BE NO \( x \)'s, \( y \)'s, or \( z \)'s IN YOUR ANSWER, AND ALL DERIVATIVES (including grad, curl, and div) SHOULD BE CARRIED OUT IN FULL. YOUR ANSWER SHOULD BE AN ALGEBRAIC EXPRESSION OF \( n, \vec{r}, \) AND \( r \). Symbols like “+”, “-”, “(”, “)” etc. are of course allowed. Numbers (“1”, “2”, “3”, etc.) are allowed. If you are still unclear what I want, ASK ME.

1. grad (\( \frac{1}{r} \))
2. grad (\( \frac{1}{r^n} \))
3. curl\( \vec{r} \)
4. div (\( \frac{\vec{r}}{r^n} \))
5. \( \nabla^2 (\frac{1}{r^n}) \)
Space for work.
Problem 2 (10 points): Let $S$ be the restriction to the positive octant of the surface of revolution obtained by revolving the curve $y = \cos(z)$, $0 \leq z \leq \pi/2$ about the $z$ axis.

1. Find a parameterization of $S$.

2. Find an equation for the tangent plane to $S$ at the point $(\frac{1}{2}, \frac{1}{2}, \frac{\pi}{4})$.

3. Find the surface area of $S$. You may use the identity

$$\int_0^1 \sqrt{1 + u^2} \, du = \frac{\sqrt{2}}{2} - \frac{1}{2} \log(\sqrt{2} - 1).$$
Space for work.
Problem 3 (10 points): Evaluate the integral
\[
\int_C \vec{F} \cdot d\vec{r}
\]
where \( \vec{F}(x, y) = (e^x + x^2y, ey - xy^2) \) and \( C \) is the circle \( x^2 + y^2 = 25 \) oriented clockwise.
Problem 4 (10 points):
Suppose we know that the vector field
\[ \vec{F} = (Axy - y^2 + y) \vec{i} + (3x^2 - Bxy + x) \vec{j} \]
is conservative where \( A \) and \( B \) are constants.

1. Find the values of \( A \) and \( B \).

2. Find a scalar function \( f(x, y) \) such that \( \vec{F} = \nabla f \).

3. Evaluate the line integral
\[ \int_C \vec{F} \cdot d\vec{r} \]
where \( C \) is the curve from \((1,1)\) to \((\frac{1}{2}, 2)\) given by \( xy = 1 \).

4. Evaluate the line integral
\[ \int_C (6xy - y^2 + y)dx + (3x^2 - 3xy + x)dy \]
where \( C \) is the curve from \((1,1)\) to \((\frac{1}{2}, 2)\) given by \( xy = 1 \).
Space for work.
Problem 5 (10 points). Say whether the following statements are true (T) or false (F). You may assume that all functions and vector fields have derivatives of all orders everywhere in their domain. You do not need to give reasons; this problem will be graded by answer only.

1. The divergence of $\nabla \times \mathbf{F}$ is zero, for every $\mathbf{F}$.
2. In a simply connected region, $\int_C \mathbf{F} \cdot d\mathbf{r}$ depends only on the endpoints of $C$.
3. If $\nabla f = \mathbf{0}$, then $f$ is a constant function.
4. If $\nabla \times \mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is a constant vector field.
5. If $\mathbf{F} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ satisfies $Q_y = P_x$, then $\mathbf{F}$ is conservative.