Problem 1 (2 points each): Let \( \vec{r} \) be the vector field

\[
\vec{r} = x \vec{i} + y \vec{j} + z \vec{k},
\]

let \( r = |\vec{r}| \), and let \( n \) be an integer. Compute the following and express your answer in terms of \( n, \vec{r}, \) and \( r \). **CREDIT WILL BE GIVEN FOR THE ANSWERS ONLY, YOU MUST EXPRESS YOUR ANSWER IN TERMS OF \( n, \vec{r}, \) AND \( r \) TO RECEIVE CREDIT.**

1. \( \text{grad}(r) \)
2. \( \text{grad}(r^n) \)
3. \( \text{div}\vec{r} \)
4. \( \text{div}(r^n\vec{r}) \)
5. \( \nabla^2(r^n) \)
Space for work.
Problem 2 (10 points): Let $S$ be the surface of revolution obtained by revolving the curve $z = y^3$, $0 \leq y \leq 1$ about the $y$ axis.

1. Find a parameterization of $S$.

2. Find the surface area of $S$.

3. Evaluate the integral

$$
\iint_S \cos^2(\theta) \sqrt{1 + 9y^4} \, dS
$$
Problem 3 (10 points):
Suppose we know that the vector field
\[ \vec{F} = A x^3 y^2 z \hat{i} + (z^3 + B x^4 y z) \hat{j} + (3 y z^2 - x^4 y^2) \hat{k} \]
is conservative.

1. Find the values of \( A \) and \( B \).

2. Find a scalar function \( f(x, y, z) \) such that \( \vec{F} = \nabla f \).
**Problem 4 (10 points):** Let $C$ be the oriented curve in the $xy$ plane from $(0, 0)$ to $(\pi, 0)$ given by $y = \sin(x)$. Let

$$\vec{F} = (x + y) \hat{i} + (2x - e^{\sqrt{y}}) \hat{j}.$$ 

Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ (hint: do not try to evaluate the integral directly).
**Problem 5 (10 points).** Say whether the following statements are true (T) or false (F). You may assume that all functions and vector fields have derivatives of all orders everywhere in their domain. You do not need to give reasons; this problem will be graded by answer only.

1. If the domain of a function $f$ is open and connected, then $\text{div} (\text{grad}(f)) = 0$. 
   
2. If $\vec{F} = P\vec{i} + Q\vec{j}$, and $\vec{F}$ is conservative, then $Q_x - P_y = 0$. 
   
3. If the domain of $\vec{F}$ is connected and simply connected, and $\text{curl} \vec{F} = 0$, then $\vec{F}$ is conservative. 
   
4. If $\int_C \vec{F} \cdot d\vec{r} = 0$ for every closed curve $C$ in the domain of $\vec{F}$, then there is a function $f$ such that $\vec{F} = \text{grad}(f)$. 
   
5. Every function $f(x, y, z)$ is the divergence of some vector field.