Problem 1 (25 points): Consider the curve with the parameterization
\[ r(t) = ti + t^2 j + tk \]
for \(-\infty < t < \infty\).

1. (15 points.) Compute the curvature \( \kappa(t) \) as a function of \( t \).

2. (5 points.) What is the limit of \( \kappa(t) \) as \( t \) approaches infinity? What is the limit of \( \kappa(t) \) as \( t \) approaches minus infinity?

3. (5 points.) For what value of \( t \) is \( \kappa(t) \) maximum?
Problem 2 (25 points):

1. **15 points.** Find the length of the curve

\[ \mathbf{r}(t) = e^t \mathbf{i} + e^t \sin t \mathbf{j} + e^t \cos t \mathbf{k} \]

from \( t = 0 \) to \( t = T \) where \( T \) is any positive number.

2. **10 points.** Reparameterize \( \mathbf{r}(t) \) with respect to arclength measured from the point where \( t = 0 \) in the direction of increasing \( t \).
Problem 3. In each problem below, select the correct answer; you do not need to show work; no partial credit will be given.

Let \( \mathbf{r}(t) \) be a vector valued function. Let \( \mathbf{r}', \mathbf{r}'' \), and \( \mathbf{r}''' \) denote \( \frac{d\mathbf{r}}{dt}, \frac{d^2\mathbf{r}}{dt^2}, \) and \( \frac{d^3\mathbf{r}}{dt^3} \) respectively.

1. 15 points. \( \frac{d}{dt}[(\mathbf{r} \times \mathbf{r}') \cdot \mathbf{r}'''] \) is given by
   (a) \( (\mathbf{r}' \times \mathbf{r}''') \cdot \mathbf{r}''' \)
   (b) \( (\mathbf{r}' \times \mathbf{r}''') \cdot \mathbf{r} + (\mathbf{r} \times \mathbf{r}') \cdot \mathbf{r}''' \)
   (c) \( (\mathbf{r} \times \mathbf{r}') \cdot \mathbf{r}''' \)
   (d) 0
   (e) None of the above.

2. 15 points. \( \frac{d}{dt}|\mathbf{r}(t)| \) is given by:
   (a) \( |\mathbf{r}'(t)| \)
   (b) \( \frac{\mathbf{r} \cdot \mathbf{r}'}{|\mathbf{r}|} \)
   (c) \( 2\mathbf{r} \cdot \mathbf{r}' \)
   (d) 0
   (e) None of the above.
Problem 4 (25 points): Evaluate the integral

\[ \int_C (x^2 + y^2)(xdy - ydx) \]

where \( C \) is the curve that bounds the “pie shaped” region in the \( xy \)-plane given by \( \{ x \geq 0, y \geq 0, x^2 + y^2 \leq 1 \} \) and \( C \) is oriented in the counter-clockwise direction.