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1. (20 marks). Provide a short answer in the box to each question.

1. (5 marks) Compute $\text{div}(x^2y\mathbf{i} + e^y\sin x\mathbf{j} + e^{xy}\mathbf{k})$

2. (5 marks) Compute $\text{curl}(\cos x^2\mathbf{i} - y^3z\mathbf{j} + xz\mathbf{k})$
3. (5 marks). Given \( \mathbf{F} = \frac{x}{x^2+y^2} \mathbf{i} + \frac{y}{x^2+y^2} \mathbf{j} + z^2 \mathbf{k} \), let \( D \) be the domain of \( \mathbf{F} \). Consider the following four statements

(I) \( D \) is connected

(II) \( D \) is disconnected

(III) \( D \) is simply connected

(IV) \( D \) is not simply connected

Choose one of the following:

(a) (II) and (III) are true

(b) (I) and (III) are true

(c) (I) and (IV) are true

(d) (II) and (IV) are true

(e) Not enough information to determine
4. (5 marks). True or False? If the speed of a particle is constant then the acceleration of the particle is zero. If your answer is True, provide a reason. If your answer is False, provide a counter example.
2. (10 marks). Given the 2 dimensional vector field \( \mathbf{F} = P\mathbf{i} + Q\mathbf{j} \) shown in page 7 choose one of the following answers.

1. Assuming that the vector field in the picture is a force field, the work done by the vector field on a particle moving from point \( A \) to \( B \) along the given path is:
   (a) Positive
   (b) Negative
   (c) Zero
   (d) Not enough information to determine.

2. Which statement is the most true about the line integral \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \):
   (a) \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} > 0 \)
   (b) \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 0 \)
   (c) \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} < 0 \)
   (d) Not enough information to determine.

3. \( \nabla \cdot (\mathbf{F}) \) at point \( N \) (in the picture) is:
   (a) Positive
   (b) Negative
   (c) Zero
   (d) Not enough information to determine.

4. \( Q_x - P_y \) at point \( Q \) is:
   (a) Positive
   (b) Negative
   (c) Zero
   (d) Not enough information to determine.
5. Assuming that \( \mathbf{F} = Pi + Qj \), which of the following statements is correct about \( \frac{\partial P}{\partial x} \) at point \( D \)?

(a) \( \frac{\partial P}{\partial x} = 0 \) at \( D \).
(b) \( \frac{\partial P}{\partial x} > 0 \) at \( D \)
(c) \( \frac{\partial P}{\partial x} < 0 \) at \( D \)
(d) The sign of \( \frac{\partial P}{\partial x} \) at \( D \) can not be determined by the given information.
3. (10 marks) A particle of mass $m = 1$ has position $\mathbf{r}(0) = \mathbf{j}$ and velocity $\mathbf{v}_0 = \mathbf{i} + \mathbf{k}$ at time $t = 0$. The particle moves under a force

$$\mathbf{F}(t) = \mathbf{j} - \sin t \mathbf{k}$$

where $t$ denotes time.

1. (3 marks). Find the position $\mathbf{r}(t)$ of the particle as a function of $t$.

2. (3 marks). Find the position $\mathbf{r}(t_1)$ of the particle when it crosses the plane $x = \pi/2$ for the first time at $t_1$.

3. (4 marks). Determine the work done by $\mathbf{F}$ in moving the particle from $\mathbf{r}(0)$ to $\mathbf{r}(t_1)$. 


4. (10 marks). Let
\[ \mathbf{F}(x, y, z) = < y, -z, e^{-y^2} + y^{1+x^2} + \cos(z) > \]

Let \( S \) be the portion of the surface which consists of two parts:

1. The portion of the paraboloid \( y^2 + x^2 = 4(z + 1) \) satisfying \( 0 \leq z \leq 3 \) and
2. The portion of the sphere \( x^2 + y^2 + z^2 = 4 \) satisfying \( z > 0 \) and is oriented upward.

Compute
\[ \iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S}. \]
5. (10 marks). Consider the curve $C$ given by

$$r(t) = \frac{1}{3}t^3\mathbf{i} + \frac{1}{\sqrt{2}}t^2\mathbf{j} + t\mathbf{k} \quad \infty < t < \infty$$

1. (3 marks). Find the unit tangent $T(t)$ as a function of $t$.
2. (3 marks). Find the curvature $\kappa(t)$ as a function of $t$.
3. (1 marks). Evaluate $\kappa(t)$ at $t = 0$.
4. (2 marks). Determine the principal normal vector $N(t)$ at the $t = 0$.
5. (1 marks). Compute the binormal vector $B(t)$ at $t = 0$. 


6. (10 marks). Let $S$ be the surface given by the equation

$$x^2 + z^2 = \sin^2 y$$

lying between the planes $y = 0$ and $y = \pi$.

1. (2 marks). Draw a picture of the surface $S$ in $\mathbb{R}^3$ including the coordinate axes.
2. (4 marks). Find a parameterization of $S$.
3. (4 marks). Evaluate the integral

$$\int \int_S \sqrt{1 + \cos^2(y)}dS.$$
7. (10 marks). Suppose the curve $C$ is the boundary of the region enclosed between the curves $y = x^2 - 4x + 3$ and $y = 3 - x^2 + 2x$. Determine the value of the line integral

$$\int_C (2xe^y + \sqrt{2 + x^2})\,dx + x^2(2 + e^y)\,dy$$

where $C$ is traversed counter-clockwise.
8. (10 marks). Consider the vector field

\[ \mathbf{F}(x, y, z) = \frac{x \mathbf{i} + y \mathbf{j} + z \mathbf{k}}{[x^2 + y^2 + z^2]^{3/2}}. \]

1. (3 marks). Compute \( \text{div}(\mathbf{F}) \).

2. (2 marks). Let \( S_1 \) be the sphere given by

\[ x^2 + (y - 2)^2 + z^2 = 9 \]

oriented outwards. Compute \( \iint_{S_1} \mathbf{F} \cdot \mathbf{n} \, dS \).

3. (2 marks). Let \( S_2 \) be the sphere given by

\[ x^2 + (y - 2)^2 + z^2 = 1 \]

oriented outwards. Compute \( \iint_{S_2} \mathbf{F} \cdot \mathbf{n} \, dS \).

4. (3 marks). Are your answers to (2) and (3) the same or different? Give a mathematical explanation of your answer.
9. (10 marks). Given:

\[ \mathbf{F} = (x - a)ye^x \mathbf{i} + (xe^x + z^3) \mathbf{j} + byz^2 \mathbf{k} \]

where \( a \) and \( b \) are some real numbers. Suppose that it is known that \( \mathbf{F} \) is conservative.

1. (2 marks). Determine \( a \) and \( b \).

2. (3 marks). Find a potential \( f(x, y, z) \) such that \( \nabla f = \mathbf{F} \).

3. (2 marks). Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is the curve defined

\[ \mathbf{r}(t) = (t, \cos^2 t, \cos t), \quad 0 \leq t \leq \pi \]

4. (3 marks). Evaluate the line integral

\[ I = \int_C (x + 1)ye^x dx + (xe^x + z^3) dy + 4yz^2 dz, \]

where \( C \) is the same curve as in part (3). Note that the coefficient 4 appearing on the right most term is not a typo!