Math215/255 Section 104 Quiz 3 (15 Minutes)

October 13, 2017

Instructions: Answer ALL questions.

Question One:

Consider the following system of first order ODEs

\[ \frac{dy_1}{dt} = 3y_1(t) - 4y_2(t) \]
\[ \frac{dy_2}{dt} = y_1(t) - y_2(t) \]

(a) Find the general solution of the system.
(b) Use the initial conditions \( y_1(0) = 1 \) and \( y_2(0) = 1 \) to find the constants in your solution.
(c) Sketch the solution of the system near \((0,0)\).

Show details of your solution.
\[ \vec{u}(t) = (1) e^t + (2) e^{-t} \]
Consider the following system of first order ODEs
\[
\frac{dy_1}{dt} = y_1(t) - 2y_2(t) \\
\frac{dy_2}{dt} = 3y_1(t) - 4y_2(t)
\]

(a) Compute the eigenvalues and eigenvectors of the system.
(b) Find all equilibria (steady state solution) of the system and classify them.
(c) Use the eigenvalues and eigenvectors to sketch the solution of the system near (0, 0).
Show details of your solution.

Let \[ A = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \]

For the eigenvalues,
\[ \lambda^2 + 3\lambda + 2 = 0 \]
\[ \lambda_1 = -2 \text{ and } \lambda_2 = -1 \]

For the eigenvector, \[ \lambda_1 = -2, \]
\[ \begin{pmatrix} 3 & -2 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]
\[ \vec{v}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \]

For \[ \lambda_2 = -1, \]
\[ \begin{pmatrix} 2 & -2 \\ 3 & -3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]
\[ \vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]
For the steady state solutions, we need to set
\[ \dot{\mathbf{y}}(t) = \begin{pmatrix} y_1'(t) \\ y_2'(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]
and solve
\[ \begin{align*}
    y_1 - 2y_2 &= 0 \\
    3y_1 - 4y_2 &= 0
\end{align*} \]
\[ \Rightarrow \begin{align*}
    3(2y_2) - 4y_2 &= 0 \\
    6y_2 - 4y_2 &= 0 \\
    2y_2 &= 0 \\
    y_2 &= 0 \\
    \Rightarrow y_1 &= 0
\end{align*} \]
\[ (y_1, y_2) = (0, 0) \]

is the only equilibrium of the system.

It is a stable node.