Question One:

(1) This slope field indicates that the associated differential equation has which form

\[ y(t)' = f(t) \]

(a) \( y(t)' = f(t) \)
(b) \( y(t)' = f(y) \)
(c) \( y(t)' = f(t, y) \)
(d) None of the above

(2) Which of the following differential equation matches the slope field

\[ y(t)' = t - y \]

(a) \( y(t)' = t - y \)
(b) \( y(t)' = y/t \)
(c) \( y(t)' = ty \)
(d) \( y(t)' = y \)
Question Two:

Consider the initial value problem (IVP)

\[
\frac{dy}{dt} = 3 - 2t - y, \quad y(0) = 1,
\]

(a) To approximate this differential equation using Euler's method, what difference formula would you get?

(b) With \( h = 0.1 \), use the difference formula to approximate \( y(0.2) \).

Show the details of your solution.

\( \text{Euler method is given by}
\]

\[
Y_{n+1} = Y_n + h \cdot f(t_n, Y_n)
\]

But \( f(t_n, Y_n) = 3 - 2t_n - Y_n \)

\[
Y_{n+1} = Y_n + h \cdot (3 - 2t_n - Y_n)
\]

\[
Y_{n+1} = (1-h)Y_n + (3-2t_n)h
\]

\( \text{formula} \)

\( \text{when} \ n=0, \)

\[
Y_0 = (1-h)Y_0 + (3-2t_0)h
\]

\[
= (1-0.1)1 + (3-2(0))0.1
\]

\[
y_0 = 1 - 0.9 + 0.3 = 1.2
\]

\[
y_1 = (1-0.9)1.2 + (3-2(0.1))0.1
\]

\[
y_1 = 1.36
\]

\[
y(0.2) \approx 1.36
\]
Question Three:
Consider the following first order ODE

\[(3xy + y^2) \, dx + (x^2 + xy) \, dy = 0\]

(a) Is the equation exact?
(b) If yes, find the general solution. Otherwise, find an integrating factor \(h(x)\) and use it to find the general solution of the equation.
(c) Find the constant in the general solution using \(y(1) = 1\).
Show the details of your solution.

\[M(x, y) = 3xy + y^2, \quad N(x, y) = x^2 + xy\]

\[M_y = 3x + 2y \neq 2x + y = N_x\]

\[\therefore M_y \neq N_x\]

\[\therefore \text{The equation is not exact.}\]

To find \(h(x)\), we need to solve

\[\frac{dh(x)}{dx} = \left(\frac{M_y - N_x}{N}\right) h(x) = \left[\frac{(3x + 2y) - (2x + y)}{x^2 + xy}\right] h(x)\]

\[= \left[\frac{6x + y}{x(x + y)}\right] h(x) = \left(\frac{1}{x}\right) h(x)\]

\[\frac{dh(x)}{dx} = \left(\frac{1}{x}\right) h(x)\]

\[\int h(x) \, dx = \int \left(\frac{1}{x}\right) h(x) \, dx\]

Integrating,

\[\ln h(x) = \ln h(x) + C_1\]

\[h(x) = C_2 e^{\ln h(x)} = C_2 x\]

\[h(x) = x\]

\[\text{Let, } C_2 = 1\]
Multiply equation \( h(x) = x \) through equation \( 1 \)

\[
(3x^2y + x^3y^2) \, dx + (x^3 + x^2y) \, dy = 0
\]

\[
M(x,y) = 3x^2y + x^3y^2, \quad N(x,y) = x^3 + x^2y
\]

Let \( \psi(x,y) = C \)

\[
\frac{\partial \psi}{\partial x} = 3x^2y + x^3y^2
\]

Integrating, \( \psi(x,y) = x^3y + \frac{1}{2}x^2y^2 + \gamma_1(y) \)

\[
\frac{\partial \psi}{\partial y} = x^3 + x^2y
\]

Integrating, \( \psi(x,y) = x^3y + \frac{1}{2}x^2y^2 + \gamma_2(x) \)

Equating \( 2 \) and \( 3 \), we have

\[
\gamma_1(y) = \gamma_2(x) = 0
\]

- General solution

\[
\psi(x,y) = x^3y + \frac{1}{2}x^2y^2
\]

The general solution is \( x^3y + \frac{1}{2}x^2y^2 = C \)

\[
y(1) > 1 \quad \Rightarrow \quad 1 + \frac{1}{2} = C
\]

\[
\Rightarrow \quad C = 3/2
\]

\[
x^3y + \frac{1}{2}x^2y^2 = 3/2
\]