Math215/255 Section 104 Quiz 1 (15 Minutes)

Name: Solution............................................ Student Number.........................................................

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Question 1:
Solve the IVP

\[ y' = \frac{7}{5}(y-3)^{2/7}, \quad y(1) = 3 \]

Is the solution unique near \((t, y) \equiv (1, 3)\)? Justify your answer.

\[ \frac{dy}{dt} = \frac{7}{5}(y-3)^{2/7} \]

\[ \int \frac{dy}{(y-3)^{2/7}} = \int \frac{7}{5} \, dt + c_1 \]

\[ \int (y-3)^{-2/7} \, dy = \frac{7}{5} t + c_1 \]

\[ (y-3)^{5/7} = \frac{7}{5} t + c_1 \]

Multiplying through by \(5/7\), we have

\[ (y-3)^{5/7} = t + c_2 \quad (c_2 = \frac{5}{7} c_1) \]

\[ y(1) = 3 \Rightarrow 0 = 1 + c_2 \Rightarrow c_2 = -1 \]

\[ (y-3)^{5/7} = t - 1 \]

Observe that \(y(t) = 3\) is also a solution.

\[ \frac{dy}{dt} = 0 \]

Substitute \(y = 3\) into (1), we also have \(\frac{dy}{dt} = 0\)

\[ y(1) = 3 \quad -y(t) = 3 \quad \text{satisfy the IVP}. \]

Solution not unique near \((1, 3)\):
Question 2:
Solve the IVP
\[ \frac{dy}{dt} + \frac{2y}{t} = \frac{\sin(t)}{t^2}, \quad y(\pi) = \frac{2}{\pi^2} \]
Show the details of your solution.

\[ \phi(t) = \frac{2}{t}, \quad \phi(t) = \frac{\sin(t)}{t^2} \]

\[ h(t) = e^{\int \phi(t) \, dt} = e^{\int \frac{\sin(t)}{t^2} \, dt} = e^{\int \frac{\sin(t)}{t^2} \, dt} = e^{-t^2} \]

Multiply through \( \phi \) by \( t^2 \).

\[ t^2 \frac{dy}{dt} + 2ty = \frac{\sin(t)}{t} \]

\[ \frac{d}{dt} \left( t^2y(t) \right) = \sin(t) \]

\[ \int d \left( t^2y(t) \right) = \int \sin(t) \, dt + C_1 \]

\[ t^2y(t) = -\cos(t) + C_1 \]

\[ y(t) = -\cos(t) + \frac{C_1}{t^2} \]

\[ y(\pi) = \frac{2}{\pi^2} \]

\[ \Rightarrow \quad \frac{2}{\pi^2} = -\cos(\pi) + \frac{C_1}{\pi^2} \quad \Rightarrow \quad C_1 = 2 - 1 \quad \Rightarrow \quad \frac{C_1}{\pi^2} = 1 \]

\[ y(t) = \frac{1}{t^2} \left( 1 - \cos(t) \right) \]